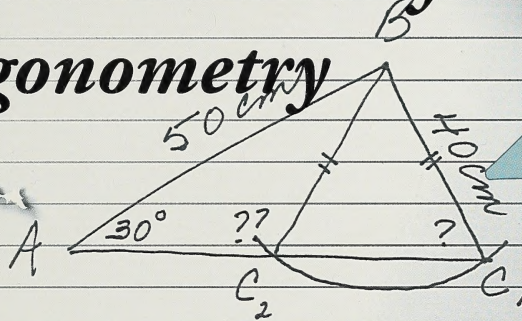


Pure MATHEMATICS

Module 6

Coordinate Geometry and Trigonometry

20



$$\sin C = \frac{c \sin A}{a}$$

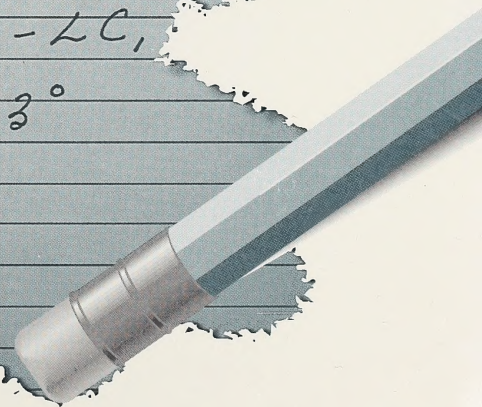
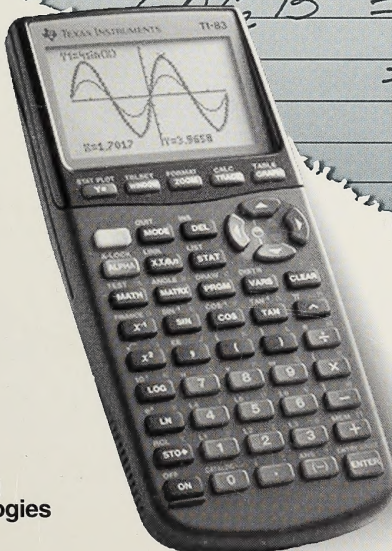
$$= \frac{50 \sin 30}{40}$$

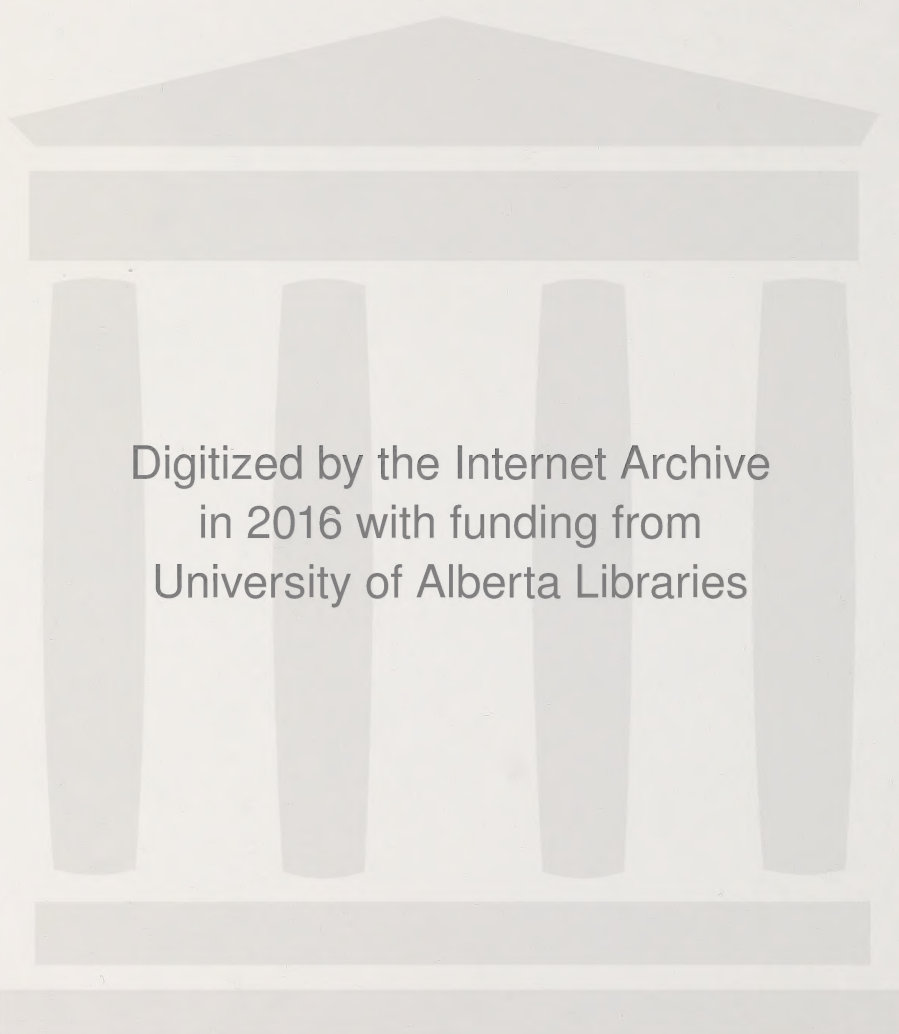
$$\angle C = 38.7^\circ$$

$$\therefore \angle C_1 = 38.7^\circ$$

$$\angle C_2 B = 180 - \angle C_1$$

$$= 141.3^\circ$$





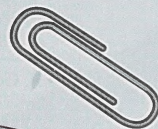
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Pure MATHEMATICS 20

Module 6 Coordinate Geometry and Trigonometry

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Pure Mathematics 20
Student Module Booklet
Module 6
Coordinate Geometry and Trigonometry
Learning Technologies Branch
ISBN 0-7741-1552-1

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Parents	
General Public	
Other	



The Learning Technologies Branch has an Internet site that you may find useful.
The address is as follows:

<http://ednet.edc.gov.ab.ca/ltb>

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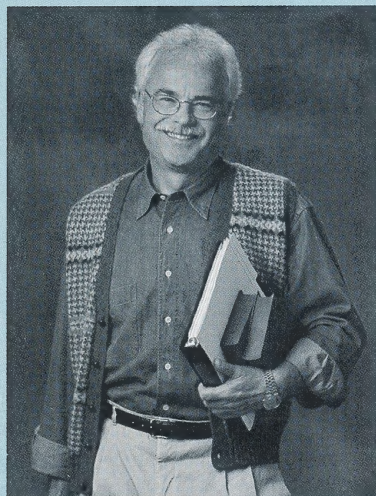
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Welcome



Welcome to Module 6. We hope you'll enjoy your study of Coordinate Geometry and Trigonometry.

Pure Mathematics 20

Module 1
Consumer Mathematics

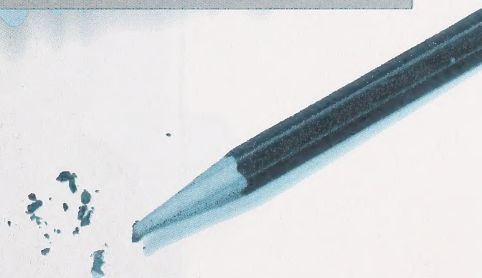
Module 2
Equations and Inequalities

Module 3
Quadratics

Module 4
Functions

Module 5
Mathematical Reasoning and Geometry

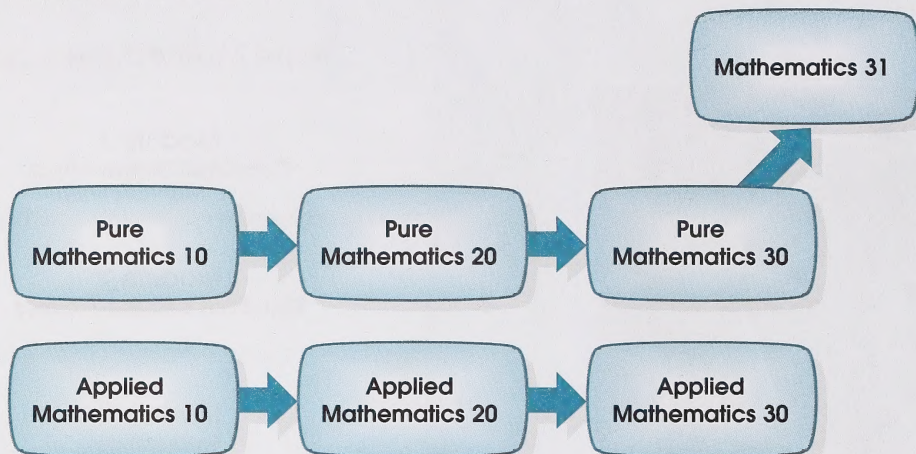
Module 6
Coordinate Geometry and Trigonometry



Pure Mathematics 20 contains six modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.

High School Mathematics Courses

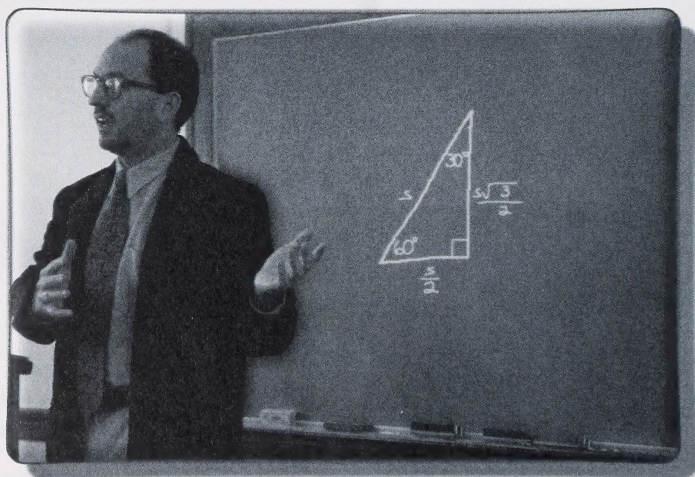
Pure Mathematics 20 is the second course in the Pure Mathematics 10–20–30 sequence of courses. Many students who take the Pure Mathematics 10–20–30 will also choose to take Mathematics 31. Another sequence of courses is Applied Mathematics 10–20–30.



How Do the Sequences Differ?

Each sequence of courses is designed for students with different mathematical strengths and interests. The Pure Mathematics 10–20–30 sequence is intended for students who are strong in mathematical theory and algebra. The Applied Mathematics 10–20–30 sequence is better suited to students who prefer to solve mathematical problems using numerical reasoning or geometry.

Each sequence of courses is designed for students with different career plans. Students who plan to take courses such as commerce, engineering, physics, and engineering technologies in post-secondary education will need to take Pure Mathematics 30 in high school. Some of these students will also need to take Mathematics 31.

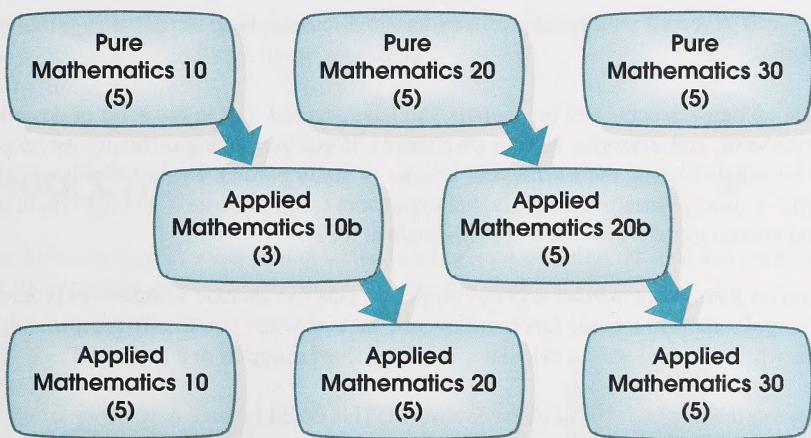


Can I Transfer to the Applied Mathematics Sequence of Courses?

You should be aware that the applied and pure mathematics courses do have some topics in common; other topics are specific to either applied or pure mathematics courses.

Applied Topics	Common Topics	Pure Topics
• linear programming • data tables and trends • design and layout • metric and imperial measure • data presentation • vectors and matrices • periodic, fractal, and recursive patterns • financial decision making • costing and design problems	• spreadsheets • line segments and linear graphs • scaling • triangles • surveys • financial mathematics • quadratic functions • circle geometry • the bell curve	• irrational numbers • exponents • polynomial and rational expressions • mathematical expectation • growth patterns • linear and non-linear systems • operations on functions • mathematical reasoning • exponential and logarithmic functions • conics • combinations • trigonometric functions

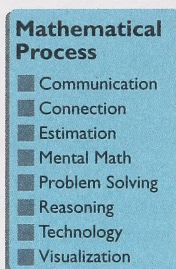
If you want to transfer from the Pure Mathematics 10–20–30 sequence to the Applied Mathematics 10–20–30 sequence at a future time, you won’t have to repeat the topics that are common to pure mathematics and applied mathematics. If you decide to transfer to the Applied Mathematics sequence of courses after successfully completing Pure Mathematics 10, you may take the 3-credit course called Applied Mathematics 10b or you may take the 5-credit course called Applied Mathematics 10. If you decide to transfer after successfully completing Pure Mathematics 20, you may take the 5-credit course called Applied Mathematics 20b. The two bridging courses are shown in the following diagram:



Mathematical Processes

Throughout the course you will be expected to do the following:

- communicate mathematically
- connect mathematical ideas to everyday experiences and to concepts in other disciplines
- develop and use estimation strategies
- develop and use mental math strategies
- develop and use problem-solving strategies
- reason and justify your thinking
- select and use appropriate technologies as tools to solve problems
- use visualization to assist in processing information, making connections, and solving problems



This icon shows which mathematical process you are expected to perform.

In order to develop these mathematical processes more fully, you are encouraged to ask someone who is also taking Pure Mathematics 20 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.

You are also encouraged to record your mathematical ideas in a journal. Here are some suggestions for organizing your journal:

- Keep a section on new concepts and procedures you have learned. Get in the habit of describing new concepts, procedures, and strategies in your own words. If you are having difficulty, write your questions in your journal before discussing them with your teacher or study partner. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, or organizational charts) to help you connect mathematical ideas.
- Keep a section on your mathematical accomplishments. This can include solutions to problems that you are proud of solving. It can also include landmark events, such as when you finally grasped a difficult concept (an “aha!”) or when you first used a calculator or spreadsheet program in a new way.
- Keep a section on mathematics in the everyday world. This could include newspaper articles, cartoons, and descriptions of careers that require a mathematical background.

Resources You Will Need

In order to complete Pure Mathematics 20 successfully, you will need to have access to the following resources, which can be purchased through the Learning Resources Distributing Centre:

- the *MATHPOWER™ 11* textbook, Western Edition, published by McGraw-Hill Ryerson (1999)
- a graphing calculator

Any of the following graphing calculators may be used: *Texas Instruments* TI-83, TI-83 Plus, TI-82, TI-86, TI-89, TI-92, or TI-92 Plus; *Casio* CFX-9850Ga-Plus, CFX-9850G, CFX-9800G, or FX-9700 series; or *Sharp* EL-9600c, EL-9600, EL-9300, or EL-9200. **Note:** Some of these calculators are no longer commercially manufactured.

- a computer and a spreadsheet program

Any of the following spreadsheet programs may be used: *ClarisWorks™ 4.0* (or higher) for Macintosh, *ClarisWorks™ 4.0* (or higher) for Windows, *Microsoft® Works 3.0* (or higher) for Windows, or *Microsoft® Access 97* and *Microsoft® Excel 97* for Windows (a part of *Microsoft® Office Professional 95* or *Microsoft® Office Professional 97*).

Note: Examples in this course show the TI-83 graphing calculator and the *ClarisWorks™ 4.0* spreadsheet program where applicable. If you have access to a videocassette recorder, you may wish to view the video *The TI-83 Graphing Calculator Video Tutor* to review some of the calculator's features. Check with your distance learning provider concerning the availability of this video.

Finally, you will need the following resources, which can be purchased locally:

- a mathematical instrument set (compass, protractor, ruler, and triangles)
- a binder in which to respond to the questions asked
- a binder for journal writing

Other Visual Cues

In addition to the Mathematical Process icon described earlier, you will find other visual cues throughout this course. Read the explanations given to discover what each icon prompts you to do.



Refer to the textbook.



Use the Internet to explore a topic.



Answer the questions in the Assignment Booklet.

Remember: Any Internet website address given in this module is subject to change.

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Course Survey

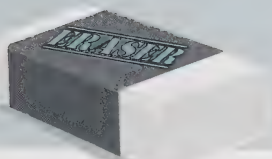
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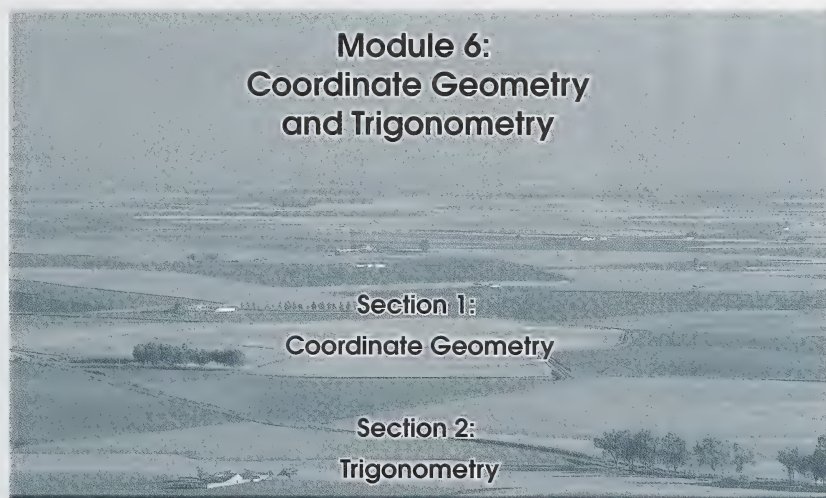




Module Overview

Have you flown over the Prairies in summer? From the sky, the roads and fields are a pattern of lines and rectangles. This is a legacy of the Dominion Lands Survey, established in 1871 to survey the land in Western Canada, formerly called Rupert's Land (primarily the present-day prairie provinces and the northern territories). Canada, which acquired these lands in 1869 from the Hudson's Bay Company, had to survey them before they could open them up to settlement. The surveyors employed a system of north-south and east-west lines, dividing the countryside into six-mile by six-mile squares, called townships. The location of these blocks of land is identified by a pair of numbers: **township** (north-south) and **range** (east-west). This rectangular survey system is very similar to the rectangular grid and coordinate system you use in coordinate geometry.

In Section 1 of this module, you will use the techniques of coordinate geometry to solve geometry problems involving lines and line segments and the circle. In Section 2, you will review and extend your skills to solve oblique triangles using trigonometry. Trigonometry, you will recall, is one of the basic mathematical tools surveyors use to get the lay of the land.



CORBIS/MICHAEL BUSSELLE

Evaluation

The document you are presently reading is called a Student Module Booklet. It will show you, step by step, what to do and how to do it.

This module, Coordinate Geometry and Trigonometry, has two sections. Within each section, your work is grouped into activities. Within the activities, there are readings and questions for you to answer. By completing these questions you will construct your own learning, discover mathematical connections, and practise or apply what you have learned. The suggested answers in the Appendix of this Student Module Booklet will provide you with immediate feedback on your progress.

At several points in this module you will be directed to the accompanying Assignment Booklet. In this module you are expected to complete two section assignments and one final module assignment. Your grading in this module is based upon the assignments that you submit for evaluation. The mark distribution is as follows:

Section 1 Assignment	60 marks
Section 2 Assignment	30 marks
Final Module Assignment	10 marks
TOTAL	100 marks



Strategies for Completing a Module

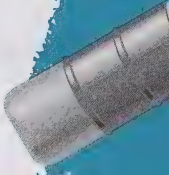
Before you begin, organize your materials: Student Module Booklet, textbook, Assignment Booklet, binder, lined paper, graph paper, graphing calculator, pens, pencils, and so on. Make sure you have a quiet area in which to work, away from distractions. Set up your time schedule.

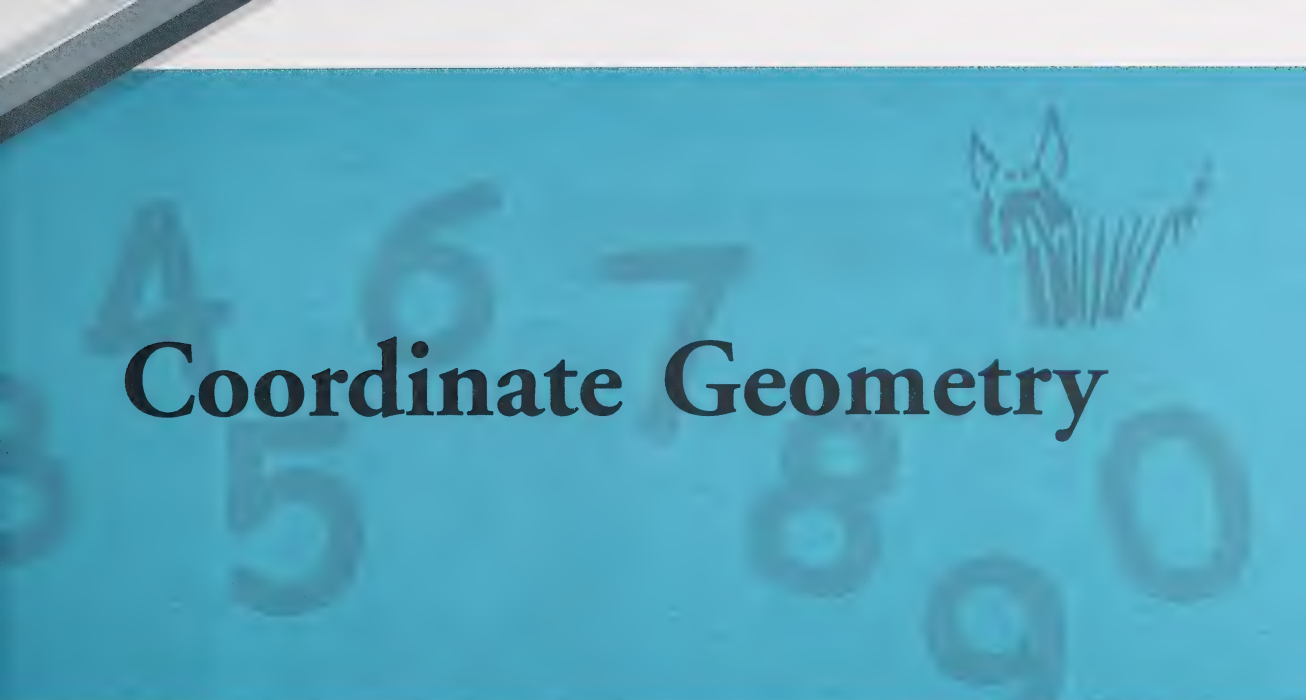
To achieve success in this module, be sure to read all your instructions carefully and work slowly and systematically through the material. Remember, it's the work you do in this Student Module Booklet that will prepare you for your assignments and final test. Try to set realistic goals for yourself each day; and once you've set them, stick to them. Submit your assignments regularly, and don't forget to review your work before handing it in. Careful work habits will greatly increase your chances for success in Pure Mathematics 20.

Good luck!



Section 1





Coordinate Geometry

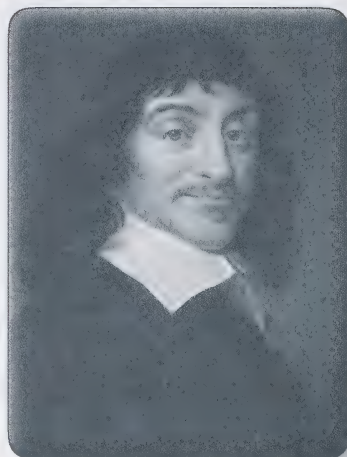
Have you ever driven in a large, unfamiliar city looking for an address? Fortunately, many North American cities use numbered streets and avenues that were surveyed along north-south and east-west lines. Finding an address in a city using a grid system is far easier than finding an address in a city that has named its streets and avenues, especially when these roads curve and do not run parallel. The grid system is an adaptation of the coordinate system used in mathematics to locate points and to position geometric figures in the plane. In mathematics, the study of geometry using a coordinate system is called coordinate geometry.

In this section, you will use coordinate geometry as a tool for investigating and verifying geometrical relationships, some of which you studied within a different context in Module 5. In particular, you will solve problems involving points, lines, and line segments; and you will be re-introduced to the circle.



Activity 1: Connecting Coordinate Geometry and Plane Geometry

René Descartes, born in late sixteenth-century France, became one of the world's most eminent philosophers, physicists, and mathematicians. Among his many achievements is the invention of coordinate geometry. He devised the system as a means of geometrically representing equations so that algebraic concepts could be represented visually. The coordinate plane also revolutionized geometry, allowing algebra to be used as a tool in formulating and analysing geometric relationships. In his memory, the coordinate plane is often referred to as the Cartesian plane.



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If you have access to the Internet, you can learn more about René Descartes at the following website:

<http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians/Descartes.html>

Coordinate systems are not unique to mathematics. Cities have used numbers to name streets and avenues, making locating addresses in towns and cities much easier.

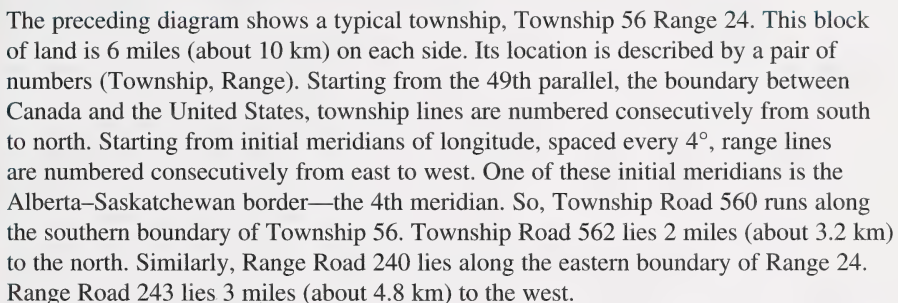


Now, have you travelled along country roads in Alberta and wondered how they are numbered? What is meant by Range Road 243 or Township Road 562?



Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization



This is a rectangular coordinate system similar to the rectangular coordinate system you will be using in this activity. In this activity, you will be investigating geometrical relationships using the coordinate plane and the tools of algebra.

Turn to page 455 of *MATHPOWER™ 11* and read “Connecting Coordinate Geometry and Plane Geometry.”

1. Answer the following questions on page 455 of the textbook:
 - a. questions a. and b. of “Explore: Use the Diagram”
 - b. questions 1 to 4 of “Inquire”

Compare your responses with those in the Appendix, Section 1: Activity 1.

To investigate geometric relationships using coordinate geometry, you will apply the following formulas:

- distance

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- midpoint

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- slope

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- slopes of perpendicular, oblique lines

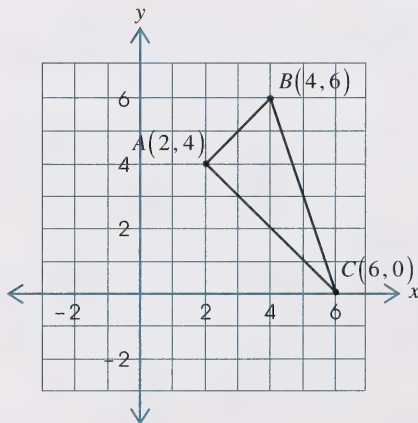
$$m_1 \times m_2 = -1$$

Example

- Use the distance formula to verify that $A(2, 4)$, $B(4, 6)$, and $C(6, 0)$ are the vertices of a right triangle.
- Determine the area of $\triangle ABC$.

Solution

- Draw the triangle on a coordinate grid.



Use the distance formula to find the length of each side.

For AB , let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (4, 6)$.

$$\begin{aligned}\therefore AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 2)^2 + (6 - 4)^2} \\ &= \sqrt{2^2 + 2^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \\ &= 2\sqrt{2}\end{aligned}$$

For AC , let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (6, 0)$.

$$\begin{aligned}\therefore AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 2)^2 + (0 - 4)^2} \\ &= \sqrt{4^2 + (-4)^2} \\ &= \sqrt{16 + 16} \\ &= \sqrt{32} \\ &= 4\sqrt{2}\end{aligned}$$

For BC , let $(x_1, y_1) = (4, 6)$ and $(x_2, y_2) = (6, 0)$.

$$\begin{aligned}\therefore BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 4)^2 + (0 - 6)^2} \\ &= \sqrt{2^2 + (-6)^2} \\ &= \sqrt{4 + 36} \\ &= \sqrt{40} \\ &= 2\sqrt{10}\end{aligned}$$

If $\triangle ABC$ is a right triangle, then the longest side, BC , is the hypotenuse.

Check and see if the side lengths satisfy the Pythagorean Theorem,

$$(BC)^2 = (AB)^2 + (AC)^2.$$

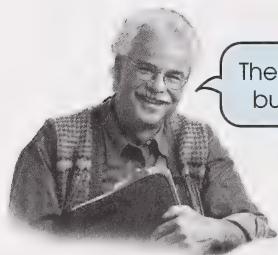
LS	RS
$(BC)^2$	$(AB)^2 + (AC)^2$
$= (2\sqrt{10})^2$	$= (2\sqrt{2})^2 + (4\sqrt{2})^2$
$= 4(10)$	$= 4(2) + 16(2)$
$= 40$	$= 8 + 32$
	$= 40$
LS	RS

Therefore, $\triangle ABC$ is a right triangle.

- b. Because the right angle is at A, sides AB and AC may be used as the base and height.

$$\begin{aligned}\therefore \text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2}AB \times AC \\ &= \frac{1}{2} \times 2\sqrt{2} \times 4\sqrt{2} \\ &= 8\end{aligned}$$

Therefore, the area of $\triangle ABC$ is 8 square units.



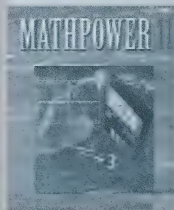
The next set of questions build on this example.

Turn to pages 456 to 459 of *MATHPOWER™ 11* and work through Examples 1 to 4.

2. Answer the following questions on pages 460 and 461 of the textbook:

- questions 1 to 4, 6, and 8 of “Practice”
- questions 14, 16, 20, and 23 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 1.



Now Try This

When you use the distance formula, you work with squares. If you can determine these squares mentally, you can estimate your answer before you enter the values into your calculator. This skill will enable you to visually check your answer to see if it is reasonable.



Having a sense of what is reasonable in a particular problem will alert you to possible formula or calculation errors!

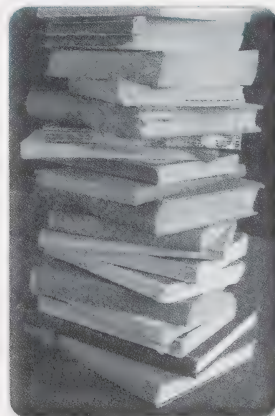
3. Turn to page 453 of *MATHPOWER™ 11* and answer questions 1 to 16 of “Mental Math: Squaring Numbers Ending in One or Two.”

Compare your responses with those in the Appendix, Section 1: Activity 1.

Looking Back

In this activity, you explored geometrical relationships using coordinate geometry, the properties of real numbers, and the tools of algebra. This, of course, is not the approach you used in Module 5. The approach to geometry in Module 5 is similar to Euclid’s approach to geometry, which is called “synthetic.” The numerical and algebraic approach to geometry used in this module is often referred to as “analytic.”

Use your research skills and explain why the terms “analytic” and “synthetic” are used to describe these approaches. Write your findings in your journal.



Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☒ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

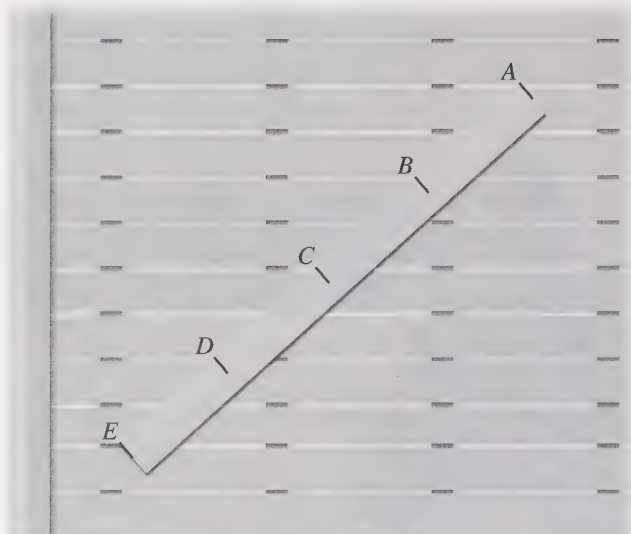
MATHPOWER



Mathematical Process

- ☒ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 2: Division of a Line Segment



A carpenter needs to divide a board into four equal parts. One of the “tricks of the trade” is to lay the board on the surface of a wooden deck. The carpenter could use the existing equally spaced and uniform decking to mark off equal divisions on the board. Simply angle the board (as shown) so that its ends are along the edges of the planking. Marks at *B*, *C*, and *D* divide the board into four equal parts. The carpenter can use this method to obtain different numbers of divisions.

In this activity, you will divide line segments in a coordinate plane into equal parts.

Turn to page 464 of *MATHPOWER™ 11* and read “Division of a Line Segment.”

1. Answer the following questions on pages 464 and 465 of the textbook:

- a. “Explore: Look for a Pattern”
- b. questions 1 to 3 of “Inquire”

Compare your responses with those in the Appendix, Section 1: Activity 2.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

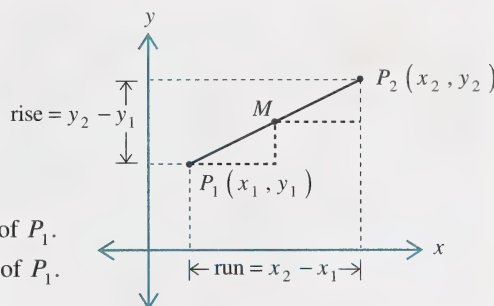
MATHPOWER



In Pure Mathematics 10, you found the midpoint of a line segment using the formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

One way you can develop the formula for the midpoint M is to first determine the run (the horizontal distance from P_1 to P_2) and then the rise (the vertical distance from P_1 to P_2).



Add one-half the run to the x -coordinate of P_1 .

Add one-half the rise to the y -coordinate of P_1 .

Therefore, the midpoint is

$$\begin{aligned} \left(x_1 + \frac{x_2 - x_1}{2}, y_1 + \frac{y_2 - y_1}{2} \right) &= \left(\frac{2x_1}{2} + \frac{x_2 - x_1}{2}, \frac{2y_1}{2} + \frac{y_2 - y_1}{2} \right) \\ &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \end{aligned}$$

You can use a similar approach when dividing a line segment into more than two equal parts.



Turn to page 465 of *MATHPOWER™ 11* and read from the red line to the red line on page 467, working through Examples 1 and 2.

2. Answer the following questions on pages 467 to 469 of the textbook:

- a. questions 5, 9, 13, 16, and 21 of “Practice”
- b. questions 24, 25, 28, 31, and 35.b. of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 2.

Now Try This

When you use the midpoint formula, you are finding the arithmetic mean of the x -coordinates of the endpoints and the arithmetic mean of the y -coordinates of the endpoints.



The next set of questions will give you additional practice in finding means mentally.

3. Turn to page 453 of *MATHPOWER™ 11* and answer questions 1 to 8 of “Mental Math: Order of Operations.”

Compare your responses with those in the Appendix, Section 1: Activity 2.

Looking Back

In this activity, you explored the division of line segments into equal parts.

Did you know there is a formula like the midpoint formula, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$, that can be used to determine the points that divide a segment into three equal parts?

• First Division Point

$$\left(\frac{2x_1+x_2}{3}, \frac{2y_1+y_2}{3}\right)$$

• Second Division Point

$$\left(\frac{x_1+2x_2}{3}, \frac{y_1+2y_2}{3}\right)$$

Work with another student who is taking Pure Mathematics 20 and use the methods you explored in this activity to develop and verify the preceding formulas. Record your efforts in your journal.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☒ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

MATHPOWER



Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 3: Distances Between Points and Lines

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☒ Visualization

Have you ever looked across a lake from one shore and wondered how far it was to the opposite shore? Do you agree that the shortest distance is the perpendicular distance from your position to the opposite shoreline?

In this activity, you will explore techniques for determining distances from points to lines and distances between parallel lines.



Turn to page 470 of *MATHPOWER™ 11* and read “Distances Between Points and Lines.”

1. Answer the following questions on page 470 of the textbook:

- a. question a. of “Explore: Use the Diagram”
- b. questions 1 to 4 of “Inquire”

Compare your responses with those in the Appendix, Section 1: Activity 3.

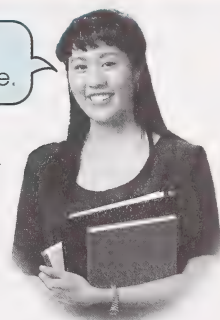
You will now investigate the process of finding the distance from a point to a line.

Turn to pages 470 to 474 of *MATHPOWER™ 11* and work through Examples 1 to 4.

2. Answer the following questions on pages 475 and 476 of the textbook:

- a. questions 1, 9, 18, 23, 32, 35, 39, and 42 of “Practice”
- b. questions 47, 48, 53, and 56 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 3.



The procedure for finding the distance between a point and a line may be summarized as follows:

Step 1: Find the slope of the given line.

Step 2: Determine the slope of the line through the given point and perpendicular to the given line.

Step 3: Use the **point-slope form** of a linear equation to find the equation of the perpendicular line.

Step 4: Determine the point of intersection of the given line and the perpendicular line by solving a **linear system**.

Step 5: Use the distance formula to determine how far the point of intersection lies from the given point.

point-slope form: the form of an equation of a line based on the slope and a point on the line

linear system: a pair of first-degree equations, each representing a line in the coordinate plane

Now Try This

The process for finding the distance between a point and a line can be adapted to find a triangle's altitude and area.

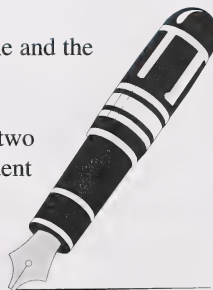
3. **a.** Graph the triangle with vertices $A(2, 3)$, $B(-2, -3)$, and $C(4, -2)$; then, determine the length of the altitude from A .
- b.** Calculate the area of $\triangle ABC$. Round your answer to the nearest hundredth.

Compare your responses with those in the Appendix, Section 1: Activity 3.

Looking Back

In this activity, you calculated the distance between a point and a line and the distance between two parallel lines.

In your journal, write the steps for calculating the distance between two parallel lines, given their equations. Show your steps to another student who is taking Pure Mathematics 20; then revise what you have done based on the student's suggestions.



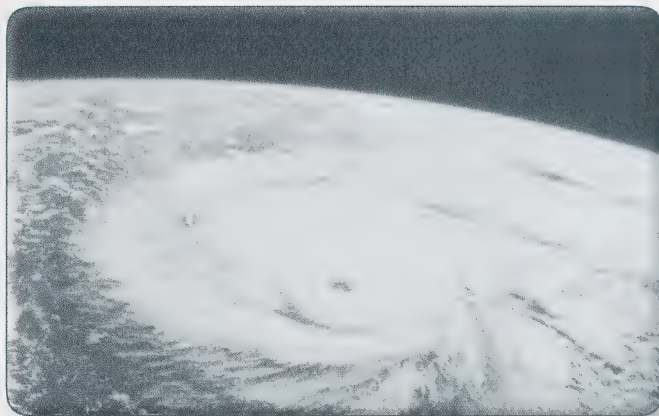
Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 4: The Circle



NASA

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

When viewed from space, a hurricane appears as a large circle of clouds with a small, clear circle at the centre, called the eye. A mature hurricane can be as large as 800 km in diameter with winds reaching speeds greater than 200 km/h. Some hurricanes have been known to have winds greater than 300 km/h.

For years, meteorologists have tracked hurricanes in an attempt to get a better understanding of their movement and behaviour. By tracking hurricanes, meteorologists can predict when a hurricane will hit land and at what magnitude, giving coastal populations ample warning.

In this activity, you will be tracking circle locations in the coordinate plane. You will determine their equations from their centres and radii, and you will investigate the intersection of lines and circles. Some of the circle properties that you covered in Module 5 of Pure Mathematics 20 will be reintroduced from a coordinate geometry perspective.

Turn to page 478 of *MATHPOWER™ 11* and read “The Equation of a Circle.”

1. Answer the following questions on page 478 of the textbook:

- a. questions a. to c. of “Explore: Draw a Graph”
- b. questions 1 to 7 of “Inquire”

Compare your responses with those in the Appendix, Section 1: Activity 4.

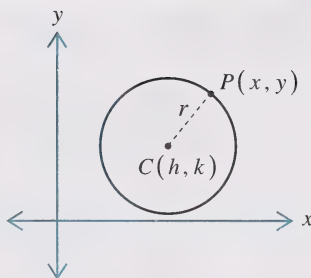
Because a **circle** is a set of points equidistant from a fixed point and because the radius is equal to the distance between the centre and any point on the circle, you can determine the equation of a circle simply by using the distance formula.

For example, for a circle, with centre $C(h, k)$ and radius r , that passes through point $P(x, y)$, its equation is as follows:

$$CP = r$$

$$\sqrt{(x-h)^2 + (y-k)^2} = r$$

$$(x-h)^2 + (y-k)^2 = r^2$$



You have just used the centre and radius to determine the **general equation of a circle**. In the next set of questions, you will be using this formula to find the equations of particular circles.

Turn to page 478 of *MATHPOWER™ 11* and read from the red line to the bottom of page 480, working through Examples 1 to 4.

2. Answer the following questions on pages 481 and 482 of the textbook:
 - a. questions 1, 12, 13, 15, 22, 27, 29, and 35 of “Practice”
 - b. questions 37.b., 40.a., 42, 43, 45, and 49 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 4.

circle: the set of all points in the plane that are equidistant from a fixed point

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

general equation of a circle: the form of an equation of a circle based on the centre and radius of the circle

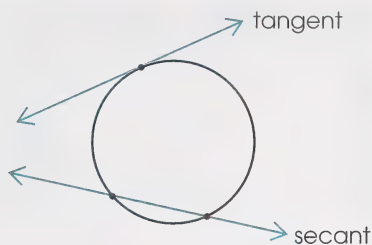
MATHPOWER™ 11



You will now investigate circles and intersecting lines.



Recall from your previous studies in mathematics that a **secant** intersects a circle at two distinct points and that a **tangent** intersects a circle at exactly one point.



Turn to page 483 of *MATHPOWER™ 11* and read “Intersections of Lines and Circles.”

3. Answer the following questions on page 483 of the textbook:

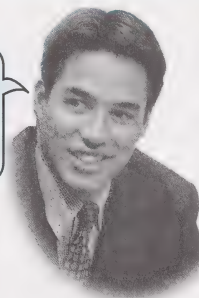
- questions a. and b. of “Explore: Draw a Graph”
- questions 1 to 3 of “Inquire”

Compare your responses with those in the Appendix, Section 1: Activity 4.

Mathematical Process

- Communication
- Connection
- Estimation
- Mental Math
- Problem Solving
- Reasoning
- Technology
- Visualization

In order to use your graphing calculator to find the points of intersection of a circle and a line, you must be able to graph a circle from its centre and radius or from its equation.



Example

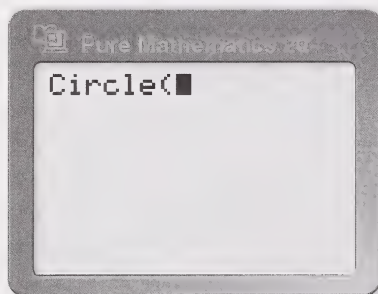
Graph $(x - 3)^2 + y^2 = 25$ on your graphing calculator.

Solution

Method 1: Using the Centre and the Radius

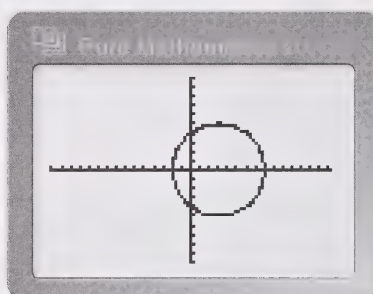
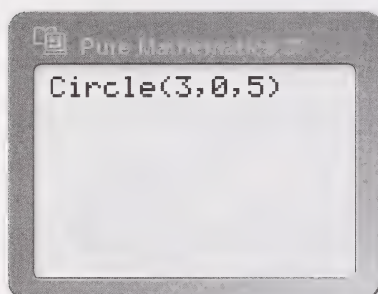
Compare $(x - 3)^2 + y^2 = 25$ with $(x - h)^2 + (y - k)^2 = r^2$. Therefore, the centre is $(3, 0)$ and the radius is 5.

Press $\boxed{2\text{nd}}$ $\boxed{[\text{DRAW}]}$ $\boxed{9}$. This pastes "Circle (" on the screen.

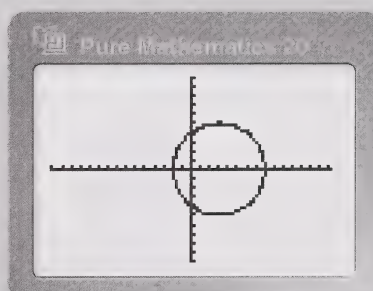
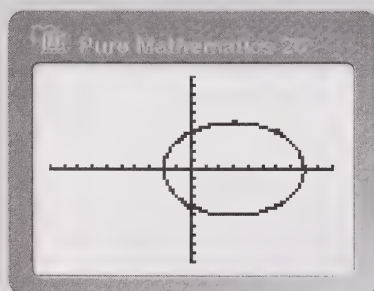


Now, enter the centre $(3, 0)$ and the radius 5.

$\boxed{3}$ $\boxed{,}$ $\boxed{0}$ $\boxed{,}$ $\boxed{5}$ $\boxed{)}$ $\boxed{\text{ENTER}}$



If you use the standard window settings, the graph will appear distorted because the scaling is not the same on both axes. To obtain the proper shape, press $\boxed{\text{ZOOM}}$ $\boxed{5}$. This will square the display.



Method 2: Using the Equation

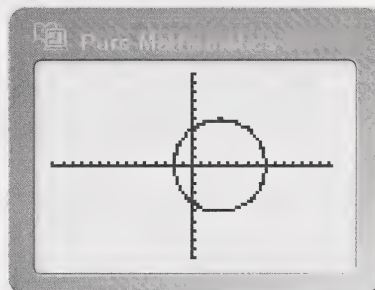
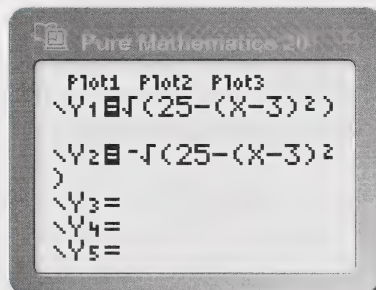
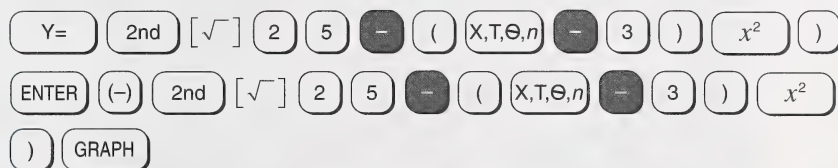
Because equations must be entered on your graphing calculator in the form $y = f(x)$, solve $(x-3)^2 + y^2 = 25$ for y .

$$(x-3)^2 + y^2 = 25$$

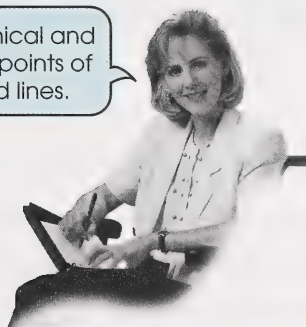
$$y^2 = 25 - (x-3)^2$$

$$y = \pm \sqrt{25 - (x-3)^2}$$

Enter $y = \sqrt{25 - (x-3)^2}$ as Y_1 and $y = -\sqrt{25 - (x-3)^2}$ as Y_2 .



Next, you will explore graphical and algebraic methods to find points of intersection of circles and lines.



Turn to pages 484 and 485 of *MATHPOWER™ 11* and work through Examples 1 and 2.

4. Answer the following questions on pages 488 and 489 of the textbook:

- a. questions 1, 6, 10, and 17 of “Practice”
- b. question 67 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 4.



In Module 5 of Pure Mathematics 20 you looked at a number of circle properties involving **chords** and tangents. You will now investigate some of the same properties from a coordinate geometry perspective.

Turn to page 486 of *MATHPOWER™ 11* and read from the red line to the red line on page 488, working through Examples 3 to 5.

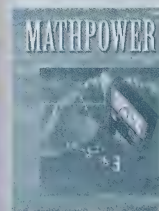
5. Answer the following questions on pages 489 to 491 of the textbook:

- a. questions 22, 27, 33, 39, 47, and 61 of “Practice”
- b. questions 68, 69, 70, 78, 87, and 91 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 1: Activity 4.



*chord of a circle: a
line segment with its
endpoints on the
circle*



Now Try This

In the following question, you will be handling tangent circles of a slightly different sort.

6. Three pennies and two loonies are arranged as shown. Adjacent coins are “tangent.”



In five moves, rearrange the coins so that the three pennies are on the left and the two loonies are on the right (as shown).



A “move” consists of sliding a penny and a loonie that form an adjacent pair to a new position, so that this pair touches at least one of the other coins.

For example, you would be allowed to make the following move. If a gap occurs after a move is made, you are not allowed to slide coins together to close the gap.



Outline the five moves required to rearrange the coins.

Compare your response with the one in the Appendix, Section 1: Activity 4.

Looking Back

In this activity, you explored the equations of circles, points of intersection between circles and lines, and tangents. You also explored the circle properties that you covered in Module 5 of Pure Mathematics 20, but this time from a coordinate geometry perspective.

In your journal, describe how coordinate geometry and plane geometry are the same and how they are different. Which perspective do you prefer working with? Explain.



Mathematical Process

- ☒ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

One of the difficulties many students have in using coordinate geometry to prove geometric conjectures is deciding how to position figures (such as triangles or quadrilaterals) in the plane. One of the rules of thumb is to locate at least one vertex at the origin and one side along the positive x -axis.

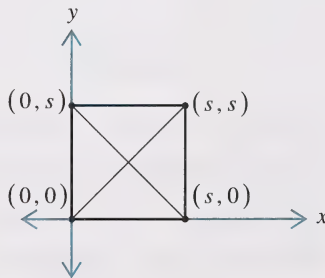
Example

Prove that the diagonals of a square intersect at right angles.

Solution

Position one of the vertices of the square at the origin, one side along the positive x -axis, and one side along the positive y -axis.

If the length of each side of the square is s units, the four vertices are $(0, 0)$, $(s, 0)$, $(0, s)$, and (s, s) .



Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

Find the slope of the segment joining $(0, 0)$ and (s, s) . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (s, s)$.

$$\begin{aligned}\text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{s - 0}{s - 0} \\ &= 1\end{aligned}$$

Find the slope of the segment joining $(s, 0)$ and $(0, s)$. Let $(x_1, y_1) = (s, 0)$ and $(x_2, y_2) = (0, s)$.

$$\begin{aligned}\text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{s - 0}{0 - s} \\ &= -1\end{aligned}$$

Because the product of the slopes of the diagonals is $(1)(-1) = -1$, the diagonals intersect at right angles.

You may wish to do the following review questions to practise the preceding techniques and others you explored throughout this section.



Turn to pages 516 to 518 of *MATHPOWER™ 11* and answer the following:

1. questions 1 to 4, 10, and 21 of “Review”
2. questions 2, 10, and 16 of “Chapter Check”

Compare your responses with those in the Appendix, Section 1: Follow-up Activities, Extra Help.

Enrichment

There is a formula that you can use to find the distance between a point, $P(x_1, y_1)$, and a line $Ax + By + C = 0$. The formula is as follows:

$$\frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

Here is how the formula was developed:

Step 1: Find the slope, m_1 , of the given line.

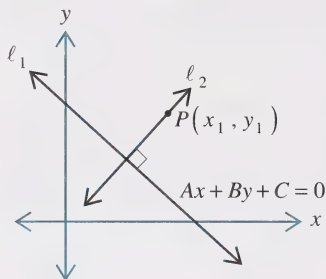
Rewrite line ℓ_1 in slope and y-intercept form,
 $y = mx + b$.

$$Ax + By + C = 0$$

$$By = -Ax - C$$

$$y = -\frac{A}{B}x - \frac{C}{B}$$

$$\therefore m_1 = -\frac{A}{B}$$



Step 2: Determine the slope, m_2 , of the line, ℓ_2 , passing through the given point and perpendicular to the given line.

The slope of the perpendicular line is the negative reciprocal of m_1 .

$$\therefore m_2 = \frac{B}{A}$$

Step 3: Use the point-slope formula to find the equation of the perpendicular line, ℓ_2 .

$$y - y_1 = m_2(x - x_1)$$

$$y - y_1 = \frac{B}{A}(x - x_1)$$

$$Ay - Ay_1 = Bx - Bx_1$$

$$Bx - Bx_1 = Ay - Ay_1$$

$$Bx - Ay = Bx_1 - Ay_1$$

Step 4: Determine the point of intersection of the given line and the perpendicular line by solving the system of equations.

$$\begin{array}{ccc} Ax + By + C = 0 & \xrightarrow{\text{Rearrange}} & Ax + By = -C \quad (1) \\ Bx - Ay = Bx_1 - Ay_1 & & Bx - Ay = Bx_1 - Ay_1 \quad (2) \end{array}$$

$$\begin{array}{rcl} A^2x + ABy & = & -AC \quad (3) : A \times (1) \\ B^2x - ABy & = & B^2x_1 - AB y_1 \quad (4) : B \times (2) \\ \hline A^2x + B^2x & = & B^2x_1 - AB y_1 - AC \quad (3) + (4) \\ (A^2 + B^2)x & = & B^2x_1 - AB y_1 - AC \\ x & = & \frac{B^2x_1 - AB y_1 - AC}{A^2 + B^2} \end{array}$$

Substitute $\frac{B^2x_1 - AB y_1 - AC}{A^2 + B^2}$ for x into (1).

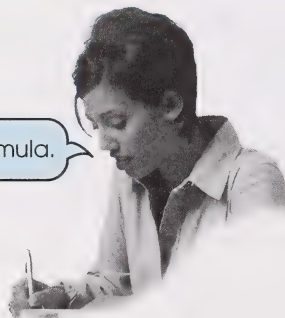
$$\begin{aligned} A \left(\frac{B^2x_1 - AB y_1 - AC}{A^2 + B^2} \right) + By &= -C \\ By &= -C - A \left(\frac{B^2x_1 - AB y_1 - AC}{A^2 + B^2} \right) \\ By &= \frac{-C(A^2 + B^2)}{A^2 + B^2} - A \left(\frac{B^2x_1 - AB y_1 - AC}{A^2 + B^2} \right) \\ By &= \frac{-\cancel{A^2}C - B^2C - AB^2x_1 + A^2By_1 + \cancel{A^2}C}{A^2 + B^2} \\ By &= \frac{-AB^2x_1 + A^2By_1 - B^2C}{A^2 + B^2} \\ y &= \frac{-ABx_1 + A^2y_1 - BC}{A^2 + B^2} \end{aligned}$$

Step 5: Use the distance formula to determine the distance, d , the point of intersection lies from the given point (x_1, y_1) .

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(\frac{B^2 x_1 - AB y_1 - AC}{A^2 + B^2} - x_1\right)^2 + \left(\frac{-AB x_1 + A^2 y_1 - BC}{A^2 + B^2} - y_1\right)^2} \\
 &= \sqrt{\left(\frac{B^2 x_1 - AB y_1 - AC - x_1(A^2 + B^2)}{A^2 + B^2}\right)^2 + \left(\frac{-AB x_1 + A^2 y_1 - BC - y_1(A^2 + B^2)}{A^2 + B^2}\right)^2} \\
 &= \sqrt{\left(\frac{\cancel{B^2 x_1} - AB y_1 - AC - \cancel{A^2 x_1} - \cancel{B^2 x_1}}{A^2 + B^2}\right)^2 + \left(\frac{-AB x_1 + \cancel{A^2 y_1} - BC - \cancel{A^2 y_1} - B^2 y_1}{A^2 + B^2}\right)^2} \\
 &= \sqrt{\left(\frac{-A^2 x_1 - AB y_1 - AC}{A^2 + B^2}\right)^2 + \left(\frac{-AB x_1 - B^2 y_1 - BC}{A^2 + B^2}\right)^2} \\
 &= \sqrt{\left(\frac{(-A)(Ax_1 + By_1 + C)}{A^2 + B^2}\right)^2 + \left(\frac{(-B)(Ax_1 + By_1 + C)}{A^2 + B^2}\right)^2} \\
 &= \sqrt{\frac{A^2 (Ax_1 + By_1 + C)^2}{(A^2 + B^2)^2} + \frac{B^2 (Ax_1 + By_1 + C)^2}{(A^2 + B^2)^2}} \\
 &= \sqrt{\frac{(A^2 + B^2)(Ax_1 + By_1 + C)^2}{(A^2 + B^2)^2}} \\
 &= \sqrt{\frac{(Ax_1 + By_1 + C)^2}{A^2 + B^2}} \\
 &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}
 \end{aligned}$$

Use absolute value because distance is non-negative.

Now, you try the formula.

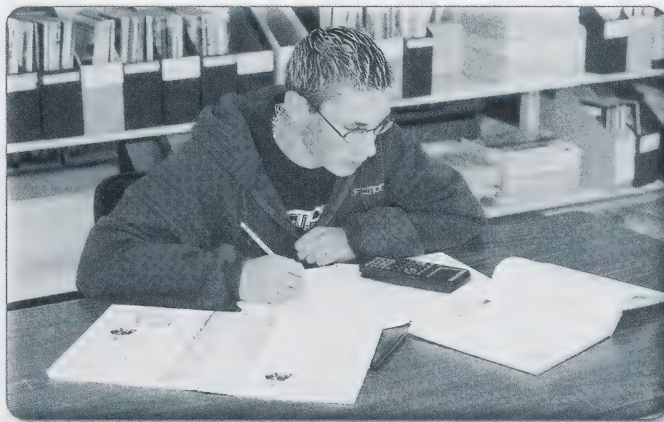


1. Find the exact distance between $(0, 4)$ and $2x - 3y = 1$.
2. Find the exact distance between $(-3, -4)$ and $2x + 3y = 10$.

Compare your responses with those in the Appendix, Section 1: Follow-up Activities: Enrichment.

Conclusion

In this section, you used coordinate geometry as a tool for investigating and verifying geometrical relationships, some of which you already knew from your previous study of plane geometry in Grade 9 mathematics. You then solved problems involving points, lines, and line segments. In particular, you divided line segments into equal parts, determined the distances from points to lines, and determined distances between parallel lines. Lastly, you examined the circle from a coordinate geometry perspective.

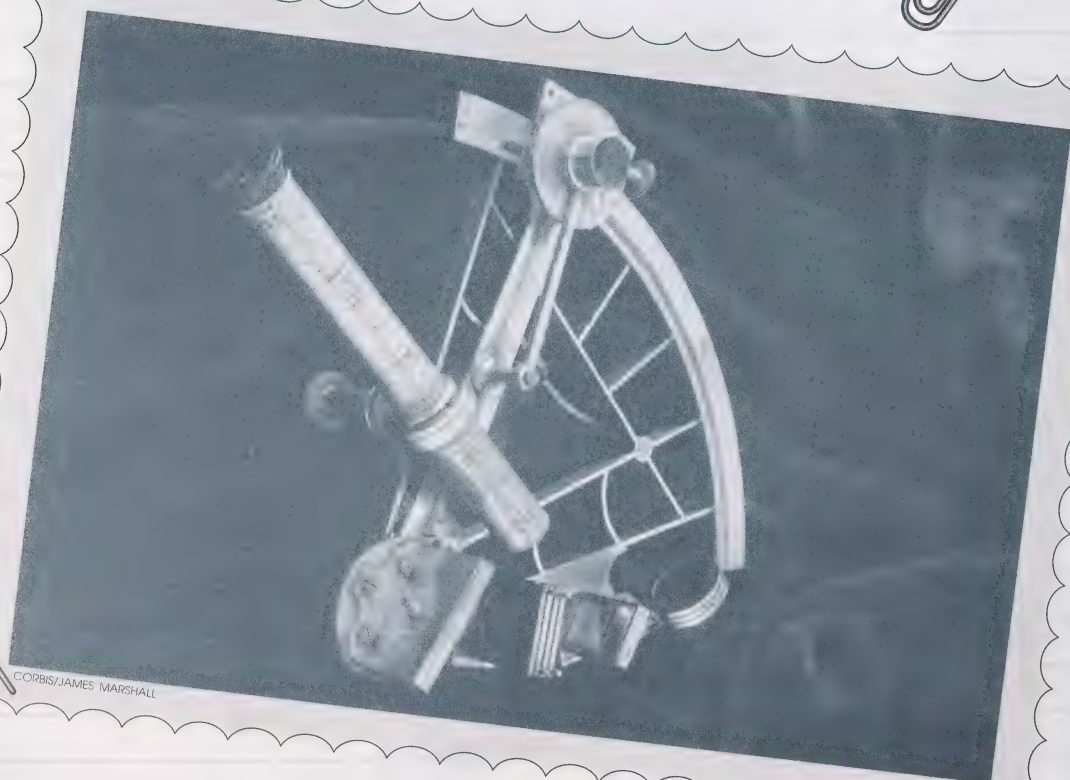


You can thank René Descartes for introducing coordinate geometry. Sir Isaac Newton, one of the founders of calculus and perhaps the world's greatest physicist and mathematician, wrote, "If I have seen further it is by standing on the shoulder of giants." One of those giants was René Descartes.

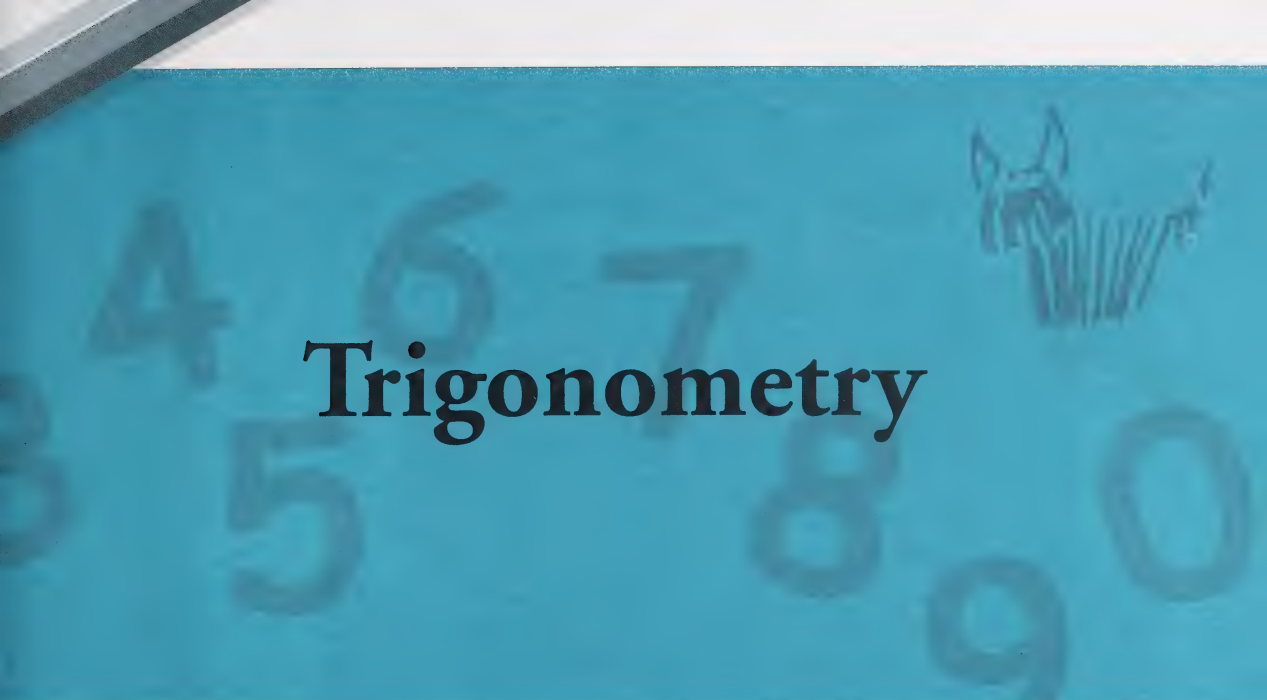
Assignment

You are now ready to complete the assignment for Section 1.

Section 2



CORBIS/JAMES MARSHALL



Trigonometry

One of the tools early surveyors employed to locate their position was the sextant. Surveyors used the sextant to measure the angle of elevation of the Sun and then calculated latitude from that measurement.

In Canada, a land of $10\,000\,000\text{ km}^2$, surveying became an integral part of the country's development. Surveyors determined boundaries; worked in the Dominion Lands Survey to prepare the Prairies for settlement; participated in geological surveys; and laid out the courses of railroads, highways, and microwave networks. Almost every element of Canada's infrastructure relied on the surveyor's careful measurements.

Trigonometry is one of the mathematical tools a surveyor uses. In this section, you will review how you can use trigonometry to solve triangles. In analysing the techniques for using the law of sines and cosines to solve oblique triangles, you will discover that solutions to some problems are not unique. In particular, you will identify situations that are ambiguous and use the Sine Law to solve for all possibilities.



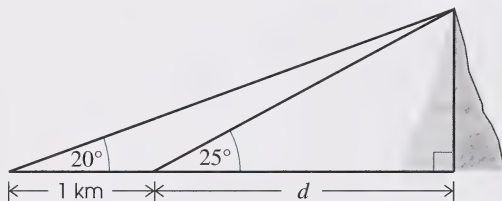
Activity 1: Reviewing Trigonometry



Among the favourite destinations of vacationers to Western Canada are the mountain parks. Many visitors associate the scale and grandeur of the mountains with the mystique of the Canadian wilderness. Indeed, part of the fascination for many travellers is first seeing the mountains appear on the horizon.

Have you wondered, when driving towards the mountains and watching them grow as you approach them, how you could calculate their distance from your position? Consider the following situation.

Suppose the angle of elevation of the peak of a distant mountain changed from 20° to 25° after driving 1 km directly towards the mountain. How would you determine the distance, d , of the mountain from your position?



Before you answer this question you will review right-triangle trigonometry.

Mathematical Process

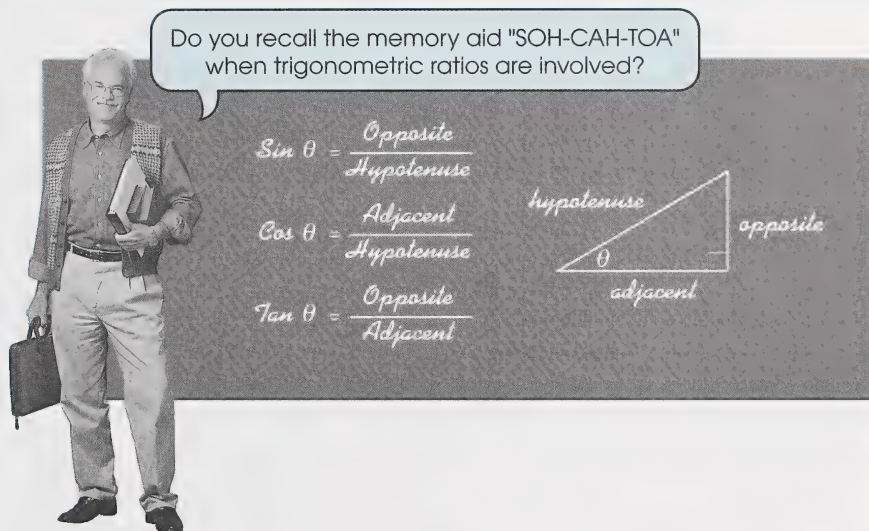
- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Turn to page 493 of *MATHPOWER™ 11* and read “Reviewing Trigonometry.”

1. Answer the following questions on page 493 of the textbook:

- a. questions a. and b. of “Explore: Use a Diagram”
- b. questions 1 to 5 of “Inquire”

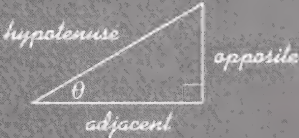
Compare your responses with those in the Appendix, Section 2: Activity 1.



Do you recall the memory aid "SOH-CAH-TOA" when trigonometric ratios are involved?

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$


These ratios are used to **solve** right triangles—finding the missing sides and angles. You will now practise using these ratios.

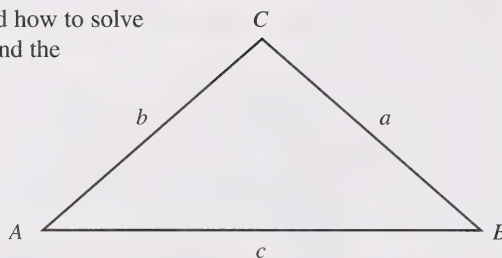
Turn to page 494 of *MATHPOWER™ 11* and read from the top of the page to the red line, working through Example 1.

2. Answer the following questions on pages 497 and 498 of the textbook:

- a. questions 2, 3, 9, 23, 25, 33, and 39 of “Practice”
- b. questions 43 and 44 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 2: Activity 1.

In Pure Mathematics 10, you discovered how to solve **oblique triangles** using the Sine Law and the Cosine Law.



oblique triangle: a triangle that is not right triangle

Sine Law	Cosine Law
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Remember:

Use the Sine Law to solve oblique triangles when given the following:

- 2 angles and 1 side
- 2 sides and the angle opposite the longer of the two sides

Use the Cosine Law to solve oblique triangles when given the following:

- 2 sides and the contained angle
- 3 sides



You will now review the application of the laws of sines and cosines.

Turn to pages 495 and 496 of *MATHPOWER™ 11* and work through Examples 2 to 4.

3. Answer the following questions on pages 497 to 499 of the textbook:

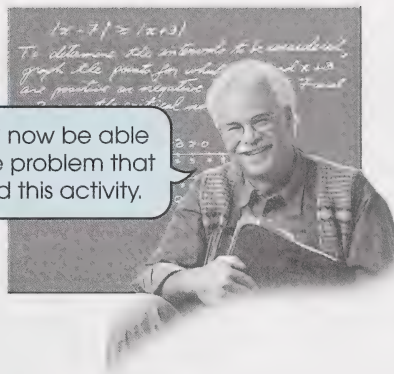
- questions 11, 16, 20, 27, 31, and 41 of “Practice”
- questions 48 and 53 of “Applications and Problem Solving”



Compare your responses with those in the Appendix, Section 2: Activity 1.

Now Try This

You should now be able to solve the problem that introduced this activity.



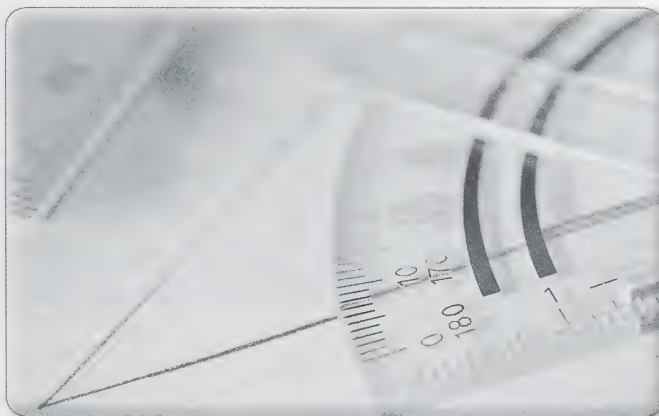
4. Determine the value of d , the distance between you and the mountain, when the angle of elevation of the peak is 25° . Round your answer to the nearest tenth of a kilometre.

Compare your response with the one in the Appendix, Section 2: Activity 1.

Looking Back

In this activity, you reviewed right-triangle trigonometry and used the Sine Law and Cosine Law to solve oblique triangles.

Review the situations when you used the Sine Law and Cosine Law to solve oblique triangles. Are there any situations that you have not yet dealt with? Describe these situations in your journal.



Mathematical Process

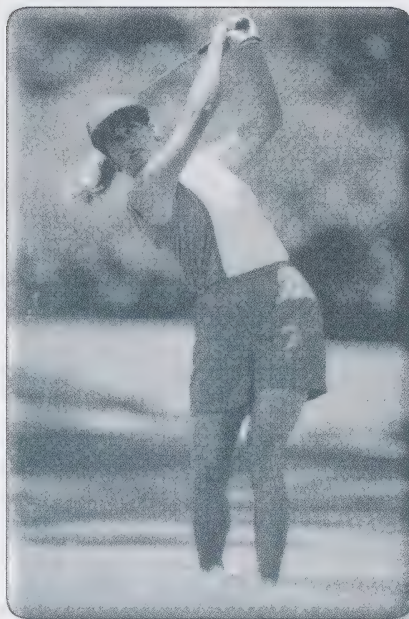
- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Mathematical Process

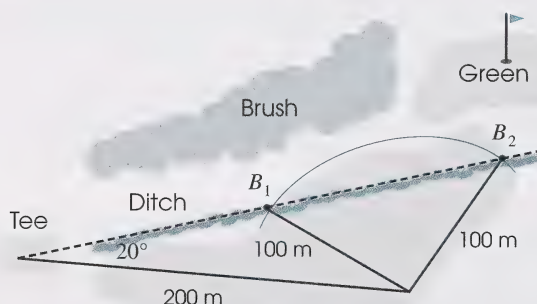
- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Activity 2: The Law of Sines—The Ambiguous Case

Anna and her friend are on the fourth fairway of a local golf course. The fourth hole, a par four, is a sharp dogleg to the left approximately 300 m from tee to pin. The first leg is bordered on the left by dense brush and a water-filled drainage ditch that must be cleared as golfers approach the green. Anna's drive carried 200 m down the fairway approximately 20° to the right of the ditch. From her lie, she was 150 m from the pin. Her second shot went only 100 m and just cleared the ditch. Where is her ball?



Without knowing whether Anna's ball travelled to the left or to the right, it could be at two different spots: B_1 or B_2 (as shown in the following diagram). As a result, your solution would be open to more than one interpretation. The situation is **ambiguous**.



In this activity, you will explore situations, like Anna's, that lead to more than one solution and apply the law of sines to find these solutions.

Mathematical Process

- ☐ Communication
- ☒ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

*ambiguous, uncertain
not uniquely defined*

Turn to page 502 of *MATHPOWER™ 11* and read “The Law of Sines: The Ambiguous Case.”



Now you can explore the ambiguous case of the law of sines.

1. Answer the following questions on page 502 of the textbook:

- a. “Explore: Draw a Diagram”
- b. questions 1 to 5 of “Inquire”



Compare your responses with those in the Appendix, Section 2: Activity 2.

Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☒ Visualization

When asked to solve an oblique triangle given two sides and an angle opposite one of the sides, drawing a diagram to scale may help you decide whether there is no solution, one solution, or two solutions.



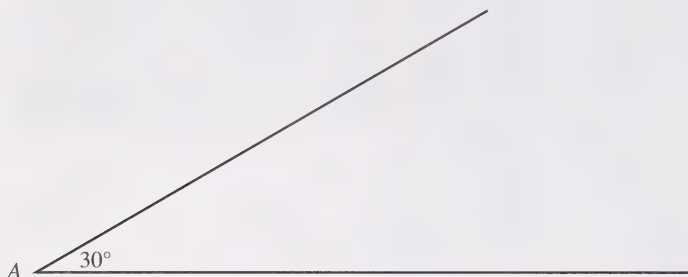
Example

Draw $\triangle ABC$, given $\angle A = 30^\circ$, $a = 5$ cm, and $c = 8$ cm.

Solution

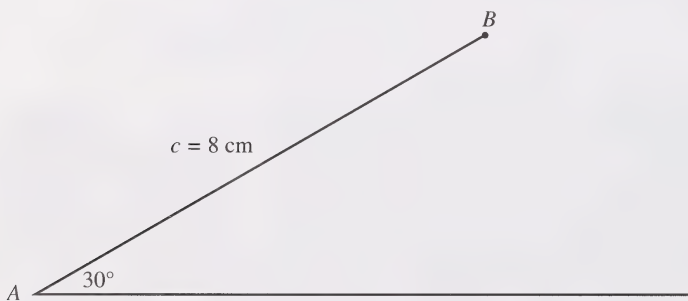
Step 1: Draw $\angle A$.

For convenience, position $\angle A$ along the base of $\triangle ABC$, with one arm horizontal (as shown). Use your protractor to draw the angle.



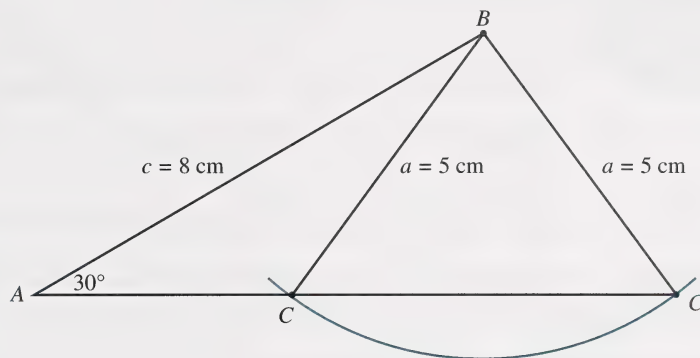
Step 2: Draw side c .

Side c will lie along one of the two arms of $\angle A$; the endpoints of side c are A and B . Again, for convenience, position side c along the upper arm of the angle. Use your ruler to locate point B 8 cm from point A .

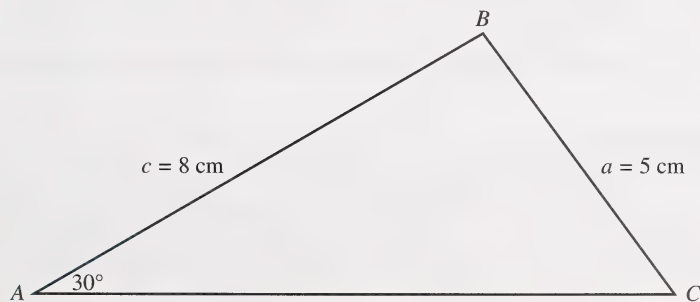
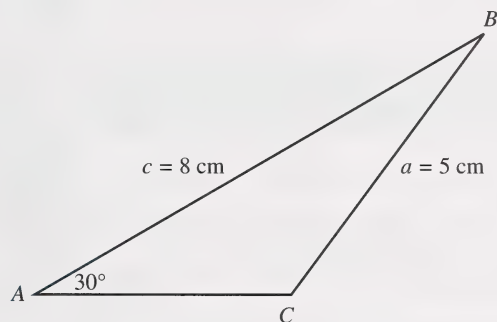


Step 3: Draw side a .

Side a is opposite $\angle A$ and lies between points B and C . To locate point C , open your compasses to a radius of 5 cm, and draw an arc using point B as the centre. This arc will intersect the lower arm of $\angle A$ at two points.



Therefore, there are two possible triangles using these elements:





You will now explore how you can obtain algebraic solutions.

Turn to page 502 of *MATHPOWER™ 11* and read from the red line to the bottom of page 509, working through Examples 1 to 4.

2. Answer the following questions on pages 510 to 512 of the textbook:

- a. questions 1, 2, 7, 8, 15, 17, and 23 of “Practice”
- b. questions 31, 32, 34, and 35 of “Applications and Problem Solving”

Compare your responses with those in the Appendix, Section 2: Activity 2.

Now Try This

Now, return to the introduction of this activity and reread the golf-ball problem.

3. Determine the possible location, as measured from the tee, along the drainage ditch, of Anna’s golf ball. Express your answer, correct to the nearest metre.



Compare your response with the one in the Appendix, Section 2: Activity 2.

Looking Back

In this activity, you discovered that the solutions to problems involving oblique triangles are not necessarily unique. You explored the conditions for no solution, one solution, and two solutions when given two sides and the angle opposite one of these sides.

In your journal, summarize the conditions necessary to yield no solution, one solution, and two solutions. Which of these conditions is properly called the ambiguous case? Illustrate your explanations with diagrams.



Mathematical Process

- ☐ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☒ Problem Solving
- ☐ Reasoning
- ☐ Technology
- ☐ Visualization

Mathematical Process

- ☒ Communication
- ☐ Connection
- ☐ Estimation
- ☐ Mental Math
- ☐ Problem Solving
- ☒ Reasoning
- ☐ Technology
- ☐ Visualization

Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

Extra Help

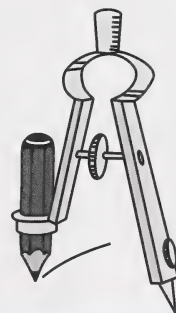
If you have access to the Internet, you can find additional examples of both the Sine Law (including the ambiguous case) and the Cosine Law at the following website:

http://www.acts.tinet.ie/trigonometryofthetrian_673.html

Many students find the ambiguous case difficult to identify and resolve. The textbook contains additional practice with constructing oblique triangles using two sides and an angle opposite one of these sides.

Turn to page 500 of *MATHPOWER™ 11* and read “Constructing Triangles Using Side-Side-Angle (SSA)””; then answer the following questions on pages 500 and 501 of the textbook:

1. questions 1 to 3 of Investigation 1, “Exploring the SSA Case”
2. questions 1 to 3 of Investigation 2, “Making Generalizations”
3. questions 1 to 3 of Investigation 3, “Applying the Concepts”



Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Extra Help.

Enrichment

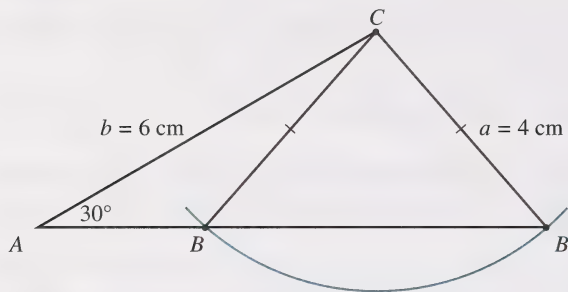
The ambiguous case occurs when you are given two sides and an angle opposite one of these sides of an oblique triangle. In Activity 2, you determined the solutions to these triangles using the Sine Law. Now, you will use the Cosine Law to determine the solutions.

Example

If in $\triangle ABC$, $\angle A = 30^\circ$, $b = 6$ cm, and $a = 4$ cm, use the Cosine Law to determine side c , to the nearest tenth of a centimetre.

Solution

Draw a diagram.



$$\begin{aligned} b \sin A &= 6 \sin 30^\circ \\ &= 3 \end{aligned}$$

Because $b \sin A < a < b$, there are two locations for point B , two triangles ABC , and two possible measures for side c , or AB .

Use the Cosine Law.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos A \\ 4^2 &= 6^2 + c^2 - 2(6)c \cos 30^\circ \\ 16 &= 36 + c^2 - 12c \cos 30^\circ \\ 0 &= c^2 - 12c \cos 30^\circ + 20 \end{aligned}$$

Use the quadratic formula to find c .

$$\begin{aligned} c &= \frac{-(-12 \cos 30^\circ) \pm \sqrt{(-12 \cos 30^\circ)^2 - 4(1)(20)}}{2(1)} \\ &\doteq 2.6 \text{ or } 7.8 \end{aligned}$$

The length of side c is about 2.6 cm or about 7.8 cm.

Turn to page 513 of *MATHPOWER™ 11* and read “The Ambiguous Case and the Law of Cosines”; then answer the following questions on page 513 of the textbook:

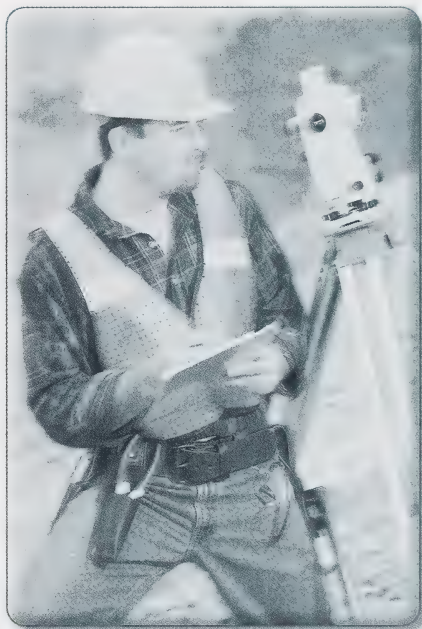
1. questions 1 to 4 of Investigation 1, “Finding the Third Side”
2. questions 1 and 2 of Investigation 2, “Solving Triangles”

Compare your responses with those in the Appendix, Section 2: Follow-up Activities, Enrichment.

Conclusion

In this section, you solved right triangles and oblique triangles. In Activity 1, you began by reviewing right-triangle trigonometry; then you solved oblique triangles using the law of sines and the law of cosines. In each instance, the solution you obtained was unique. In Activity 2, you explored situations in which the solutions were not unique. You examined the ambiguous case of the Sine Law and applied this aspect of triangle trigonometry in practical situations.

One of the practical applications of trigonometry is in surveying. Trigonometry, like the sextant, chain, and transit, was a tool used by early surveyors to help prepare this country for settlement and development. There are few areas, if any, in this nation that have not seen surveyors carry out their trade. Agriculture, transportation, mining, and urban planning all depend on the surveyor's careful measurements.



Assignment

You are now ready to complete the assignment for Section 2.

Module Summary

In Section 1, you used the techniques of coordinate geometry to solve geometric problems involving lines and line segments. You divided line segments into congruent parts, found distances between points and lines, and determined distances between parallel lines. You also explored, from a coordinate geometry perspective, some of the concepts of circle geometry you discussed in Module 5.

In Section 2, you reviewed and extended your skills in solving right triangles and oblique triangles using trigonometry.



ALBERTA AGRICULTURE, CONSERVATION AND DEVELOPMENT

The modern prairie landscape is a visual testimony to the practical nature of both coordinate systems and trigonometry. The early surveyors, working under the Dominion Lands Survey, mapped out farms and towns according to a rectangular coordinate system. If you have driven along rural grid roads or had an opportunity to see farmsteads from the air (especially in summer) you surely noticed that the fields stand out like squares imprinted on the plains.

Final Module Assignment

You are now ready to complete the final module assignment.



COURSE SURVEY FOR PURE MATHEMATICS 20

(© 1999)

After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.

Name _____ Age ☐ under 19 ☐ 19 to 40 ☐ over 40

Address _____ File No. _____

_____ Date _____

Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

☐ Yes ☐ No If yes, which style do you prefer?

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

☐ Yes ☐ No If yes, explain.

4. Were the questions and directions easy to understand?

☐ Yes ☐ No If no, explain.

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

- ☐ mainly extra help
- ☐ mainly enrichment
- ☐ a variety
- ☐ none

Did you find these activities beneficial?

- ☐ Yes ☐ No If no, explain.
-
-

6. Did you understand what was expected in the Assignment Booklets?

- ☐ Yes ☐ No If no, explain.
-
-

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

- ☐ Yes ☐ No If no, provide details.
-
-

8. This distance learning course may include an assortment of drawings, photographs, and charts.

a. Did you find the visuals in this course helpful?

- ☐ Yes ☐ No Comment on the lines below.
-
-

b. Did you find the variety of visuals in this course motivating?

- ☐ Yes ☐ No Comment on the lines below.
-
-

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

☐ Yes ☐ No Comment on the lines below.

Only students enrolled in a junior high course need to complete the following question.

10. The Student Module Booklet may have directed you to work with your learning facilitator. How well did you work as a team?

Student's comments: _____

Learning Facilitator's comments: _____

Course Content

1. Was enough detailed information provided to help you learn the expected skills and objectives?

☐ Yes ☐ No Comment on the lines below.

2. Did you find the workload reasonable?

☐ Yes ☐ No If no, explain.

3. Did you have any difficulty with the reading level?

☐ Yes ☐ No Please comment.

4. How would you assess your general reading level?

☐ poor reader ☐ average reader ☐ good reader

5. Was the material presented clearly and with sufficient depth?

☐ Yes ☐ No If no, explain.

General

1. What did you like least about the course?

2. What did you like most about the course?

Additional Comments

Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

☐ Yes ☐ No If yes, approximately how many times? _____

Did you find the staff helpful?

☐ Yes ☐ No If no, explain.

2. Were you able to fax any of your assignment response pages?

☐ Yes ☐ No If yes, comment on the value of being able to do this.

3. If you mailed your assignment response pages, how long did it take for their return?

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

☐ Yes ☐ No Please comment.

Thanks for taking the time to complete this questionnaire.
Your feedback is important to us. Please return this
questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre
and have been mailing your Assignment Booklets to ADLC,
you may return this questionnaire with the final Assignment
Booklet in the course.

Instructional Design and Development
Learning Technologies Branch
Box 4000
Barrhead, Alberta
T7N 1P4



Appendix

Glossary Suggested Answers

Glossary

ambiguous: uncertain; not uniquely defined

chord of a circle: a line segment with its endpoints on the circle

circle: the set of all points in the plane that are equidistant from a fixed point

general equation of a circle: the form of an equation of a circle based on the centre and radius of the circle

$$(x-h)^2 + (y-k)^2 = r^2$$

linear system: a pair of first-degree equations, each representing a line in the coordinate plane

median: in a triangle, the segment joining a vertex to the midpoint of the opposite side

oblique triangle: a triangle that is not a right triangle

point-slope form: the form of an equation of a line based on the slope and a point on the line

$$y - y_1 = m(x - x_1)$$

secant: a line that intersects a circle at two points

A secant of a circle contains a chord of the circle.

slope and y-intercept form of a linear equation: a linear equation written in the form $y = mx + b$, where m is the slope and b is the y-intercept

tangent: a line that intersects a circle at exactly one point

Suggested Answers

Section 1: Activity 1

1. a. Textbook questions a. and b. of “Explore: Use the Diagram,” p. 455

- a. The coordinates of home plate are $(0, 0)$; the coordinates of first base are $(27.4, 0)$; the coordinates of second base are $(27.4, 27.4)$; and the coordinates of third base are $(0, 27.4)$.
- b. The slope of the side from home plate to first base is 0; the slope from first base to second base is undefined; the slope from second base to third base is 0; and the slope from third base to home plate is undefined.

The slope of the diagonal from home plate to second base is 1; and the slope of the diagonal from third base to first base is -1 .

b. Textbook questions 1 to 4 of “Inquire,” p. 455

1. a. The product of the slope of perpendicular oblique lines is -1 . A horizontal line with slope 0 is perpendicular to a vertical line with an undefined slope.
 - b. The slopes of the diagonals are 1 and -1 . Because the product of the slopes is $1(-1) = -1$, the diagonals are perpendicular.
 - c. Yes, the sides that meet at first base are perpendicular. The side joining home plate to first base is horizontal; and the side joining second base to first base is vertical. Horizontal and vertical lines are perpendicular.
2. a. The diagonal joining home plate to second base links $(0, 0)$ to $(27.4, 27.4)$. The midpoint of this diagonal is $\left(\frac{0+27.4}{2}, \frac{0+27.4}{2}\right) = (13.7, 13.7)$. Similarly, the diagonal joining third base to first base links $(0, 27.4)$ to $(27.4, 0)$. The midpoint of this diagonal is $\left(\frac{0+27.4}{2}, \frac{27.4+0}{2}\right) = (13.7, 13.7)$.
 - b. The diagonals have a common midpoint; therefore, they bisect each other at $(13.7, 13.7)$.
3. Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (13.7, 13.7)$.

$$\begin{aligned}
 \therefore d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(13.7 - 0)^2 + (13.7 - 0)^2} \\
 &= \sqrt{13.7^2 + 13.7^2} \\
 &\doteq 19.4
 \end{aligned}$$

The distance from home plate to the point of intersection of the diagonals is approximately 19.4 m.

4. The pitcher's mound is not located at the point of intersection of the diagonals, because the pitcher's mound is only 18.4 m from home plate.

Section 1: Activity 1 (continued)

2. a. Textbook questions 1 to 4, 6, and 8 of “Practice,” p. 460

1. Use the distance formula to find the lengths of AB and BC .

For AB , let $(x_1, y_1) = (0, 0)$ and

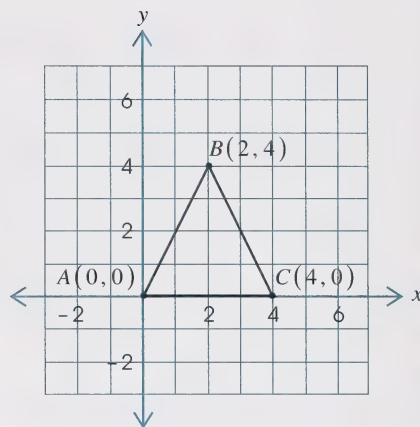
$$(x_2, y_2) = (2, 4).$$

$$\begin{aligned}\therefore AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 0)^2 + (4 - 0)^2} \\ &= \sqrt{2^2 + 4^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \\ &= 2\sqrt{5}\end{aligned}$$

For BC , let $(x_1, y_1) = (2, 4)$ and $(x_2, y_2) = (4, 0)$.

$$\begin{aligned}\therefore BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(4 - 2)^2 + (0 - 4)^2} \\ &= \sqrt{2^2 + (-4)^2} \\ &= \sqrt{4 + 16} \\ &= \sqrt{20} \\ &= 2\sqrt{5}\end{aligned}$$

Because $AB = BC$, $\triangle ABC$ is isosceles.



2. Find the slope of each side.

For side OP , let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (3, 5)$.

$$\begin{aligned}\therefore m_{OP} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 0}{3 - 0} \\ &= \frac{5}{3}\end{aligned}$$

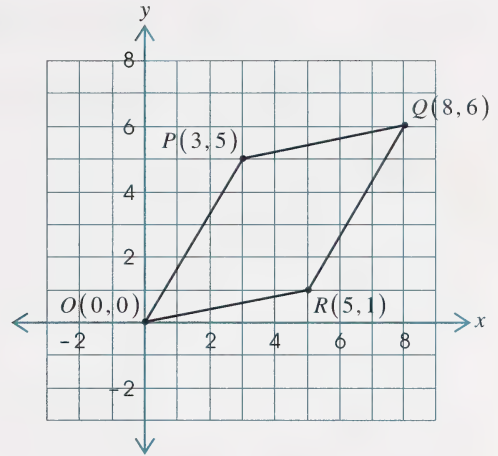
For side QR , let $(x_1, y_1) = (8, 6)$ and $(x_2, y_2) = (5, 1)$.

$$\begin{aligned}\therefore m_{QR} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 6}{5 - 8} \\ &= \frac{-5}{-3} \\ &= \frac{5}{3}\end{aligned}$$

Because $m_{OP} = m_{QR}$, $OP \parallel QR$.

For side PQ , let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (8, 6)$.

$$\begin{aligned}\therefore m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6 - 5}{8 - 3} \\ &= \frac{1}{5}\end{aligned}$$



Section 1: Activity 1 (continued)

For side OR , let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (5, 1)$.

$$\begin{aligned}\therefore m_{OR} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 0}{5 - 0} \\ &= \frac{1}{5}\end{aligned}$$

Because $m_{PQ} = m_{OR}$, $PQ \parallel OR$.

Because the opposite sides are parallel, $OPQR$ is a parallelogram.

3. The right angle appears to lie at X . So, find the slopes of sides XY and XZ .

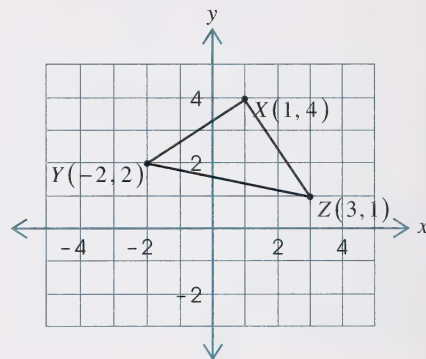
For XY , let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (-2, 2)$.

$$\begin{aligned}\therefore m_{XY} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 4}{-2 - 1} \\ &= \frac{-2}{-3} \\ &= \frac{2}{3}\end{aligned}$$

For XZ , let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (3, 1)$.

$$\begin{aligned}\therefore m_{XZ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 4}{3 - 1} \\ &= \frac{-3}{2} \text{ or } -\frac{3}{2}\end{aligned}$$

Because $m_{XY} \times m_{XZ} = \frac{2}{3} \left(-\frac{3}{2}\right) = -1$, $XY \perp XZ$; therefore, $\triangle XYZ$ is a right triangle.



4. Use the midpoint formula to find P . Let $(x_1, y_1) = (2, 6)$ and $(x_2, y_2) = (4, 10)$.

$$\begin{aligned} P &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2 + 4}{2}, \frac{6 + 10}{2} \right) \\ &= \left(\frac{6}{2}, \frac{16}{2} \right) \\ &= (3, 8) \end{aligned}$$

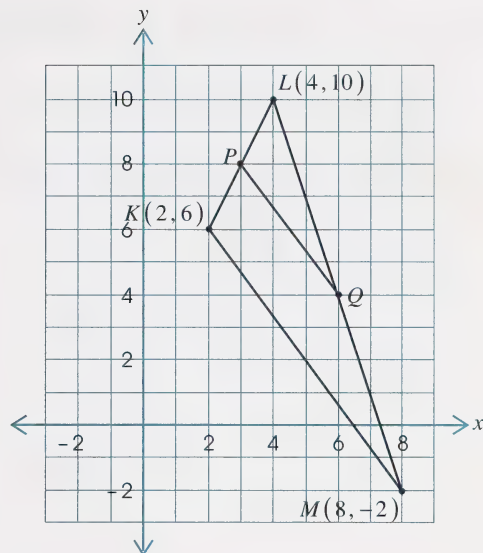
Use the midpoint formula to find Q . Let $(x_1, y_1) = (4, 10)$ and $(x_2, y_2) = (8, -2)$.

$$\begin{aligned} Q &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{4 + 8}{2}, \frac{10 + (-2)}{2} \right) \\ &= \left(\frac{12}{2}, \frac{8}{2} \right) \\ &= (6, 4) \end{aligned}$$

Now, find the slopes of PQ and KM .

For PQ , let $(x_1, y_1) = (3, 8)$
and $(x_2, y_2) = (6, 4)$.

$$\begin{aligned} \therefore m_{PQ} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 8}{6 - 3} \\ &= \frac{-4}{3} \text{ or } -\frac{4}{3} \end{aligned}$$



For KM , let $(x_1, y_1) = (2, 6)$
and $(x_2, y_2) = (8, -2)$.

$$\begin{aligned} \therefore m_{KM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 6}{8 - 2} \\ &= \frac{-8}{6} \\ &= -\frac{4}{3} \end{aligned}$$

Because $m_{PQ} = m_{KM}$, $PQ \parallel KM$.

Section 1: Activity 1 (continued)

6. Determine the midpoint of each diagonal.

For AC , let $(x_1, y_1) = (-2, -2)$

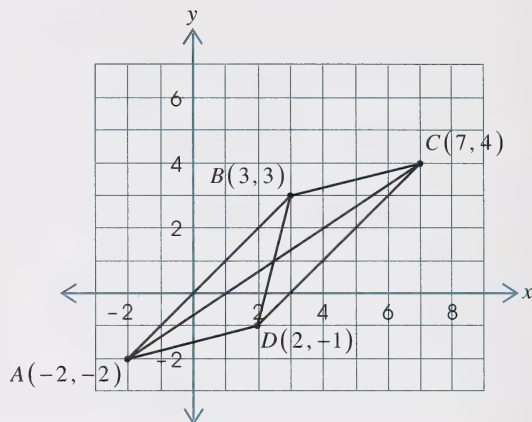
and $(x_2, y_2) = (7, 4)$.

$$\begin{aligned}\therefore \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 7}{2}, \frac{-2 + 4}{2} \right) \\ &= \left(\frac{5}{2}, \frac{2}{2} \right) \\ &= \left(\frac{5}{2}, 1 \right)\end{aligned}$$

For BD , let $(x_1, y_1) = (3, 3)$ and $(x_2, y_2) = (2, -1)$.

$$\begin{aligned}\therefore \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{3 + 2}{2}, \frac{3 + (-1)}{2} \right) \\ &= \left(\frac{5}{2}, \frac{2}{2} \right) \\ &= \left(\frac{5}{2}, 1 \right)\end{aligned}$$

Because AC and BD have a common midpoint, the diagonals bisect each other.

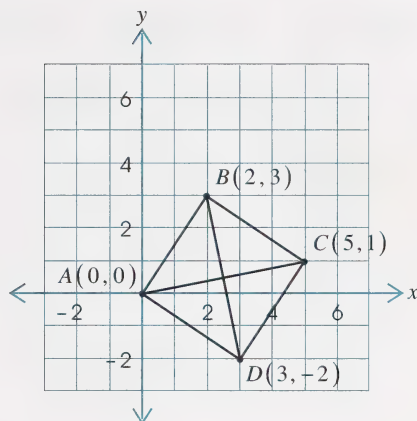


8. Find the slopes of diagonals AC and BD .

For AC , let $(x_1, y_1) = (0, 0)$

and $(x_2, y_2) = (5, 1)$.

$$\begin{aligned}\therefore m_{AC} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 0}{5 - 0} \\ &= \frac{1}{5}\end{aligned}$$



For BD , let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (3, -2)$.

$$\begin{aligned}\therefore m_{BD} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 3}{3 - 2} \\ &= \frac{-5}{1} \\ &= -5\end{aligned}$$

Because $m_{AC} \times m_{BD} = \frac{1}{5}(-5) = -1$, $AC \perp BD$.

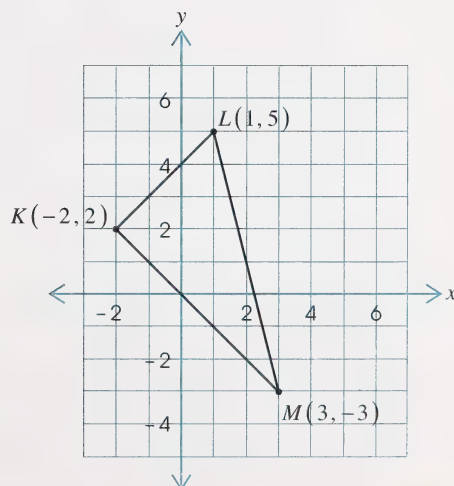
b. Textbook questions 14, 16, 20, and 23 of “Applications and Problem Solving,” pp. 460 and 461

14. a. The right angle appears to be at K . So, find the slope of sides KL and KM .

For KL , let $(x_1, y_1) = (-2, 2)$ and

$(x_2, y_2) = (1, 5)$.

$$\begin{aligned}\therefore m_{KL} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{5 - 2}{1 - (-2)} \\ &= \frac{3}{3} \\ &= 1\end{aligned}$$



Section 1: Activity 1 (continued)

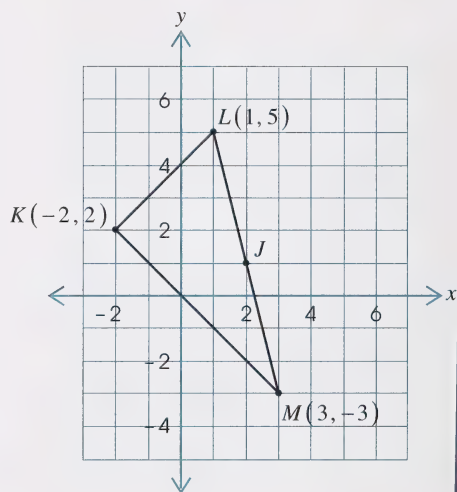
For KM , let $(x_1, y_1) = (-2, 2)$ and $(x_2, y_2) = (3, -3)$.

$$\begin{aligned}\therefore m_{KM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 2}{3 - (-2)} \\ &= \frac{-5}{5} \\ &= -1\end{aligned}$$

Because $m_{KL} \times m_{KM} = 1(-1) = -1$, $KL \perp KM$; therefore, $\triangle KLM$ is a right triangle.

- b. Because the right angle is at K , LM is the hypotenuse of $\triangle KLM$. Find the midpoint, J , of LM . Let $(x_1, y_1) = (1, 5)$ and $(x_2, y_2) = (3, -3)$.

$$\begin{aligned}J &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{1 + 3}{2}, \frac{5 + (-3)}{2} \right) \\ &= \left(\frac{4}{2}, \frac{2}{2} \right) \\ &= (2, 1)\end{aligned}$$



Find the distance from J to M . Let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (3, -3)$.

$$\begin{aligned}JM &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - 2)^2 + (-3 - 1)^2} \\ &= \sqrt{1^2 + (-4)^2} \\ &= \sqrt{1 + 16} \\ &= \sqrt{17}\end{aligned}$$

Because J is the midpoint of LM , $JM = JL = \sqrt{17}$.

Find the distance from J to K . Let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (-2, 2)$.

$$\begin{aligned}
 JK &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2 - 2)^2 + (2 - 1)^2} \\
 &= \sqrt{(-4)^2 + 1^2} \\
 &= \sqrt{16 + 1} \\
 &= \sqrt{17}
 \end{aligned}$$

Because $JK = JL = JM = \sqrt{17}$, the midpoint of the hypotenuse is the same distance from each vertex.

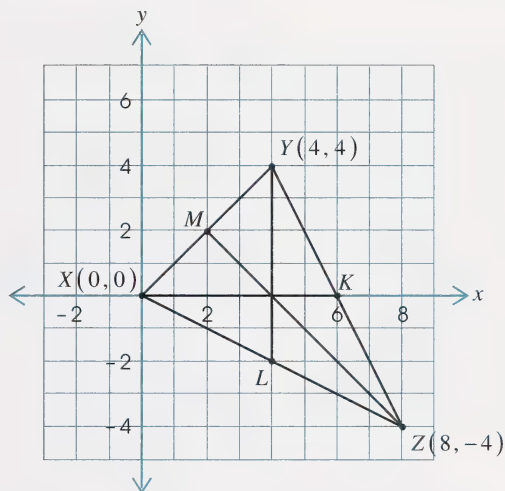
- 16. a.** Find the midpoint of each side of $\triangle XYZ$.

For K , the midpoint of YZ , let

$$(x_1, y_1) = (4, 4) \text{ and}$$

$$(x_2, y_2) = (8, -4).$$

$$\begin{aligned}
 \therefore K &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{4 + 8}{2}, \frac{4 + (-4)}{2} \right) \\
 &= \left(\frac{12}{2}, \frac{0}{2} \right) \\
 &= (6, 0)
 \end{aligned}$$



For L , the midpoint of XZ , let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (8, -4)$.

$$\begin{aligned}
 \therefore L &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{0 + 8}{2}, \frac{0 + (-4)}{2} \right) \\
 &= \left(\frac{8}{2}, \frac{-4}{2} \right) \\
 &= (4, -2)
 \end{aligned}$$

Section 1: Activity 1 (continued)

For M , the midpoint of XY , let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (4, 4)$.

$$\begin{aligned}\therefore M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{0+4}{2}, \frac{0+4}{2} \right) \\ &= \left(\frac{4}{2}, \frac{4}{2} \right) \\ &= (2, 2)\end{aligned}$$

The median from X , joining $X(0, 0)$ and $M(2, 2)$, is horizontal at $y = 0$; therefore, the equation of the median from X is $y = 0$.

The median from Y , joining $Y(4, 4)$ and $L(4, -2)$, is vertical at $x = 4$; therefore, the equation of the median from Y is $x = 4$ or $x - 4 = 0$.

The median from Z , joining $Z(8, -4)$ and $M(2, 2)$, is neither horizontal nor vertical. Therefore, find the slope ZM . Let $(x_1, y_1) = (8, -4)$ and $(x_2, y_2) = (2, 2)$.

$$\begin{aligned}m_{ZM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-4)}{2 - 8} \\ &= \frac{6}{-6} \\ &= -1\end{aligned}$$

Use the point-slope formula, $y - y_1 = m(x - x_1)$. Let $m = -1$ and $(x_1, y_1) = (8, -4)$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - (-4) &= -1(x - 8) \\ y + 4 &= -x + 8 \\ x + y - 4 &= 0\end{aligned}$$

The equation of the median from Z is $x + y - 4 = 0$.

- b. Verify that $(4, 0)$ satisfies the equation of each median.

Check the median from X , $y = 0$.

LS	RS
y	0
$= 0$	
LS	$= RS$

Check the median from Y , $x - 4 = 0$.

LS	RS
$x - 4$	0
$= 4 - 4$	
$= 0$	
LS	$= RS$

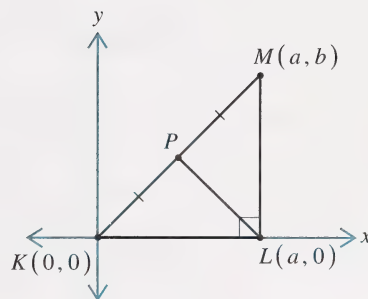
Check the median from Z , $x + y - 4 = 0$.

LS	RS
$x + y - 4$	0
$= 4 + 0 - 4$	
$= 0$	
LS	$= RS$

Therefore, $(4, 0)$ is the centroid of $\triangle XYZ$.

20. Position one vertex of the triangle at the origin and one side along the positive x -axis. Let the coordinates of the triangle be $K(0, 0)$, $L(a, 0)$, and $M(a, b)$. Locate the position of P , the midpoint of KM . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (a, b)$.

$$\begin{aligned}
 P &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{0 + a}{2}, \frac{0 + b}{2} \right) \\
 &= \left(\frac{a}{2}, \frac{b}{2} \right)
 \end{aligned}$$



Section 1: Activity 1 (continued)

Find the length of PL . Let $(x_1, y_1) = (\frac{a}{2}, \frac{b}{2})$ and $(x_2, y_2) = (a, 0)$.

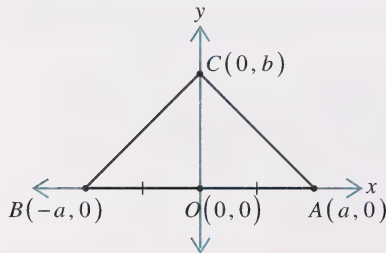
$$\begin{aligned}
 PL &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(a - \frac{a}{2}\right)^2 + \left(0 - \frac{b}{2}\right)^2} \\
 &= \sqrt{\left(\frac{a}{2}\right)^2 + \left(-\frac{b}{2}\right)^2} \\
 &= \sqrt{\frac{a^2}{4} + \frac{b^2}{4}} \\
 &= \sqrt{\frac{a^2 + b^2}{4}} \\
 &= \frac{\sqrt{a^2 + b^2}}{2}
 \end{aligned}$$

Find the length of KM . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (a, b)$.

$$\begin{aligned}
 KM &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(a - 0)^2 + (b - 0)^2} \\
 &= \sqrt{a^2 + b^2}
 \end{aligned}$$

Because $PL = \frac{\sqrt{a^2 + b^2}}{2} = \frac{KM}{2}$, the length of the median from the right angle to the hypotenuse is half the length of the hypotenuse.

23. Draw $\triangle ABC$ with coordinates $A(a, 0)$, $B(-a, 0)$, and $C(0, b)$.



Use the distance formula to show that $\triangle ABC$ is isosceles.

For BC , let $(x_1, y_1) = (-a, 0)$ and $(x_2, y_2) = (0, b)$.

$$\begin{aligned}\therefore BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - (-a))^2 + (b - 0)^2} \\ &= \sqrt{a^2 + b^2}\end{aligned}$$

For AC , let $(x_1, y_1) = (a, 0)$ and $(x_2, y_2) = (0, b)$.

$$\begin{aligned}\therefore AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(0 - a)^2 + (b - 0)^2} \\ &= \sqrt{a^2 + b^2}\end{aligned}$$

Because $BC = AC$, $\triangle ABC$ is isosceles.

Now, the midpoint of AB is $O(0, 0)$. The median from C , joining $C(0, b)$ and $O(0, 0)$, lies along the y -axis. The base of the triangle lies along the x -axis. Therefore, the median from C is perpendicular to the base AB .

3. Textbook questions 1 to 16 of "Mental Math: Squaring Numbers Ending in One or Two," p. 453

1. For 21^2 , think $20 \times 22 = 440$

$$440 + 1 = 441$$

So, $21^2 = 441$.

2. For 41^2 , think $40 \times 42 = 1680$

$$1680 + 1 = 1681$$

So, $41^2 = 1681$.

3. For 91^2 , think $90 \times 92 = 8280$

$$8280 + 1 = 8281$$

So, $91^2 = 8281$.

4. For 101^2 , think $100 \times 102 = 10\,200$

$$10\,200 + 1 = 10\,201$$

So, $101^2 = 10\,201$.

Section 1: Activity 1 (continued)

5. For 5.1^2 , think $50 \times 52 = 2600$
 $2600 + 1 = 2601$

So, $5.1^2 = 26.01$.

7. For 810^2 , think $80 \times 82 = 6560$
 $6560 + 1 = 6561$

So, $810^2 = 656\,100$.

9. For 22^2 , think $20 \times 24 = 480$
 $480 + 4 = 484$

So, $22^2 = 484$.

11. For 82^2 , think $80 \times 84 = 6720$
 $6720 + 4 = 6724$

So, $82^2 = 6724$.

13. For 9.2^2 , think $90 \times 94 = 8460$
 $8460 + 4 = 8464$

So, $9.2^2 = 84.64$.

15. For 420^2 , think $40 \times 44 = 1760$
 $1760 + 4 = 1764$

So, $420^2 = 176\,400$.

6. For 7.1^2 , think $70 \times 72 = 5040$
 $5040 + 1 = 5041$

So, $7.1^2 = 50.41$.

8. For 610^2 , think $60 \times 62 = 3720$
 $3720 + 1 = 3721$

So, $610^2 = 372\,100$.

10. For 52^2 , think $50 \times 54 = 2700$
 $2700 + 4 = 2704$

So, $52^2 = 2704$.

12. For 102^2 , think $100 \times 104 = 10\,400$
 $10\,400 + 4 = 10\,404$

So, $102^2 = 10\,404$.

14. For 6.2^2 , think $60 \times 64 = 3840$
 $3840 + 4 = 3844$

So, $6.2^2 = 38.44$.

16. For 720^2 , think $70 \times 74 = 5180$
 $5180 + 4 = 5184$

So, $720^2 = 518\,400$.

Section 1: Activity 2

1. a. Textbook question “Explore: Look for a Pattern,” p. 464

Number of Steps	Diagram	Tread Depth (m)	Riser Height (m)
1		4.0	2.4
2		2.0	1.2
3		$\frac{4}{3}$	0.8
4		1.0	0.6
n		$\frac{4}{n}$	$\frac{2.4}{n}$

b. Textbook questions 1 to 3 of “Inquire,” p. 465

1. a. $\frac{4.0}{10} = 0.4$

The tread depth would be 0.4 m.

b. $\frac{2.4}{10} = 0.24$

The riser height would be 0.24 m.

2. a. Let n be the number of steps.

$$\therefore \frac{4.0}{n} = 0.25$$

$$0.25n = 4.0$$

$$n = 16$$

There are 16 steps in the staircase.

b. $\frac{2.4}{16} = 0.15$

The riser height is 0.15 m or 15 cm.

Section 1: Activity 2 (continued)

$$3. \quad \text{a.} \quad \frac{84}{3} = 28 \qquad \frac{60}{3} = 20$$

The tread depth is 28 cm, and the riser height is 20 cm.

b. Points C and D divide AB into 3 parts.

c. The coordinates of B are $(84, 60)$; the coordinates of C are $(28, 20)$; and the coordinates of D are $(56, 40)$.

$$\text{d.} \quad \frac{84}{4} = 21 \qquad \frac{60}{4} = 15$$

The tread depth is 21 cm, and the riser height is 15 cm.

Therefore, the coordinates of K , L , and M are $(21, 15)$, $(42, 30)$, and $(63, 45)$ respectively.

2. a. Textbook questions 5, 9, 13, 16, and 21 of “Practice,” pp. 467 and 468

5. The midpoint of UV divides it into two congruent parts. Let $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (6, -1)$.

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{3 + 6}{2}, \frac{-1 + (-1)}{2} \right) \\ &= \left(\frac{9}{2}, \frac{-2}{2} \right) \\ &= \left(\frac{9}{2}, -1 \right) \end{aligned}$$

9. The horizontal distance from E to F is

$$\begin{aligned} |x_2 - x_1| &= |3 - 0| \\ &= 3 \end{aligned}$$

The horizontal distance between each division point is $\frac{3}{3}$ or 1.

The vertical distance from E to F is

$$\begin{aligned} |y_2 - y_1| &= |8 - (-1)| \\ &= 9 \end{aligned}$$

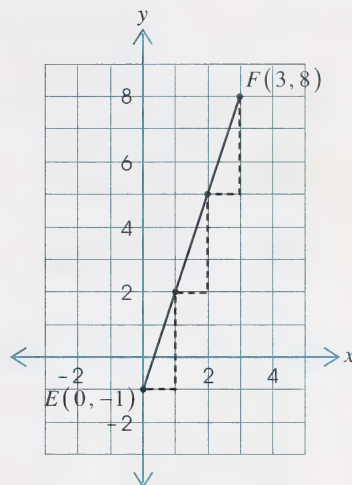
The vertical distance between each division point is

$$\frac{9}{3} \text{ or } 3.$$

Starting from E , the first division point is 1 unit to the right and 3 units up. The coordinates of this division point are $(0+1, -1+3)$ or $(1, 2)$.

The coordinates of the next division point are $(0+2(1), -1+2(3))$ or $(2, 5)$.

So, the points that divide EF into three congruent parts are $(1, 2)$ and $(2, 5)$.



13. The horizontal distance from R to S is

$$\begin{aligned} |x_2 - x_1| &= |5 - 8| \\ &= 3 \end{aligned}$$

The horizontal distance between each division point is $\frac{3}{3}$ or 1.

The vertical distance from R to S is

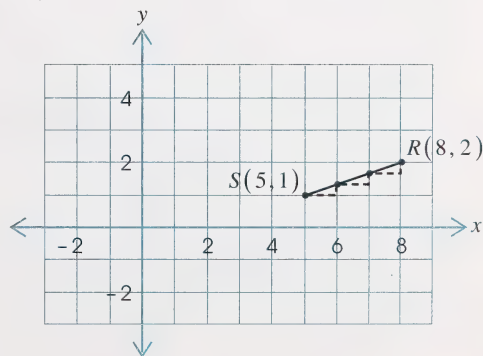
$$\begin{aligned} |y_2 - y_1| &= |1 - 2| \\ &= 1 \end{aligned}$$

The vertical distance between each division point is $\frac{1}{3}$.

Starting from S , the first division point is 1 unit to the right and $\frac{1}{3}$ of a unit up. The coordinates of this division point are $(5+1, 1+\frac{1}{3})$ or $(6, \frac{4}{3})$.

The coordinates of the next division point are $(5+2(1), 1+2(\frac{1}{3}))$ or $(7, \frac{5}{3})$.

So, the points that divide RS into three congruent parts are $(6, \frac{4}{3})$ and $(7, \frac{5}{3})$.



Section 1: Activity 2 (continued)

16. The horizontal distance from K to L is

$$\begin{aligned} |x_2 - x_1| &= |-2 - (-12)| \\ &= 10 \end{aligned}$$

The horizontal distance between each division point is $\frac{10}{4}$ or 2.5.

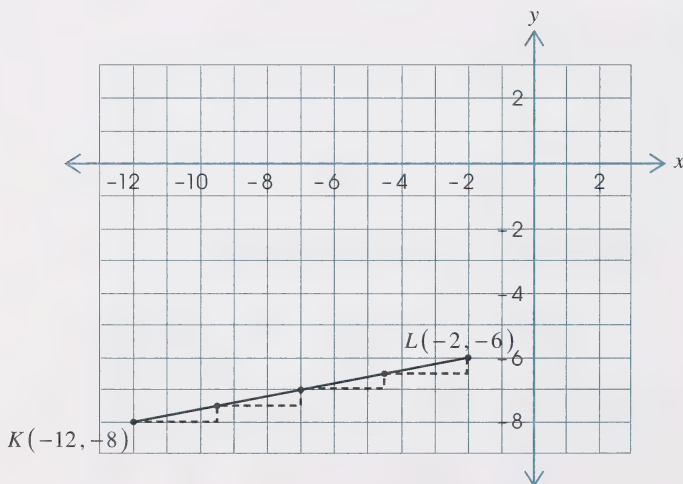
The vertical distance from K to L is

$$\begin{aligned} |y_2 - y_1| &= |-6 - (-8)| \\ &= 2 \end{aligned}$$

The vertical distance between each division point is $\frac{2}{4}$ or 0.5.

Starting from K , the first division point is 2.5 units to the right and 0.5 units up. The coordinates of this division point are $(-12 + 2.5, -8 + 0.5)$ or $(-9.5, -7.5)$.

The next two division points are $(-12 + 2(2.5), -8 + 2(0.5))$ or $(-7, -7)$ and $(-12 + 3(2.5), -8 + 3(0.5))$ or $(-4.5, -6.5)$.



So, the points that divide KL into four congruent parts are $(-9.5, -7.5)$, $(-7, -7)$, and $(-4.5, -6.5)$.

21. The horizontal distance from E to F is

$$\begin{aligned} |x_2 - x_1| &= |60 - 12| \\ &= 48 \end{aligned}$$

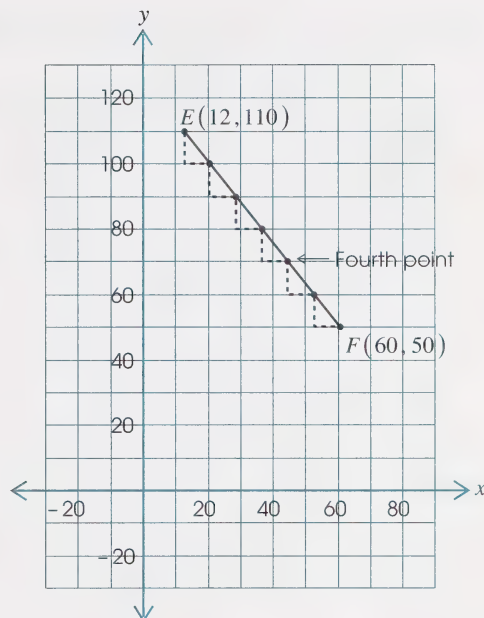
The horizontal distance between each division point is $\frac{48}{6}$ or 8.

The vertical distance from E to F is

$$\begin{aligned} |y_2 - y_1| &= |50 - 110| \\ &= 60 \end{aligned}$$

The vertical distance between each division point is $\frac{60}{6}$ or 10.

The fourth division point is to the right from point E . The coordinates of this point are $(12 + 4(8), 110 - 4(10))$ or $(44, 70)$.



- b. Textbook questions 24, 25, 28, 31, and 35.b. of “Applications and Problem Solving,” pp. 468 and 469

24. The horizontal distance between each division point is

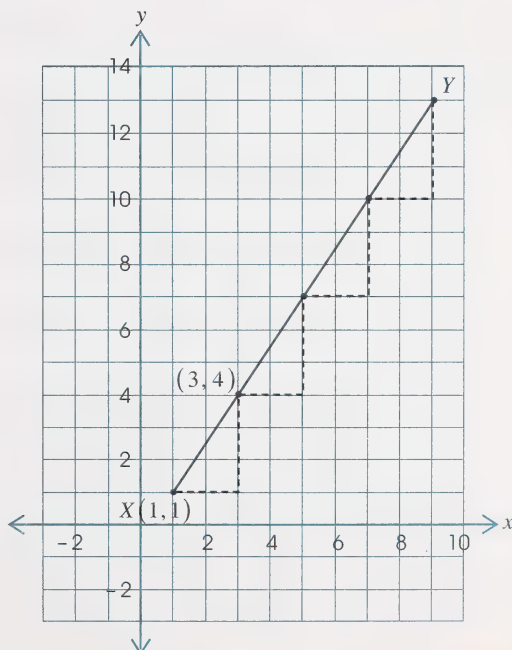
$$\begin{aligned} |x_2 - x_1| &= |3 - 1| \\ &= 2 \end{aligned}$$

The vertical distance between each division point is

$$\begin{aligned} |y_2 - y_1| &= |4 - 1| \\ &= 3 \end{aligned}$$

The endpoint Y will be 4 steps going up and to the right from point X .

The coordinates of Y are $(1 + 4(2), 1 + 4(3))$ or $(9, 13)$.



Section 1: Activity 2 (continued)

25. a. The horizontal distance from K to L is

$$\begin{aligned} |x_2 - x_1| &= |6 - (-4)| \\ &= 10 \end{aligned}$$

The horizontal distance between each division point is $\frac{10}{5}$ or 2.

The vertical distance from K to L is

$$\begin{aligned} |y_2 - y_1| &= |-1 - 19| \\ &= 20 \end{aligned}$$

The vertical distance between each division point is $\frac{20}{5}$ or 4.

The first division point lies 2 units to the right and 4 units down from K .

The coordinates of this point are $(-4 + 2, 19 - 4)$ or $(-2, 15)$.

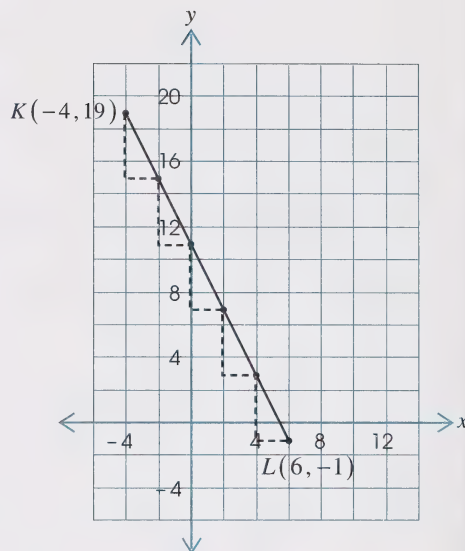
The coordinates of the other three division points are as follows:

- $(-4 + 2(2), 19 - 4(2))$ or $(0, 11)$
- $(-4 + 2(3), 19 - 4(3))$ or $(2, 7)$
- $(-4 + 2(4), 19 - 4(4))$ or $(4, 3)$

- b. The x -coordinates from least to greatest are $-4, -2, 0, 2, 4, 6$.

- c. The y -coordinates from greatest to least are $19, 15, 11, 7, 3, -1$.

- d. The sequences in 25.b. and 25.c. are arithmetic sequences. Recall that an arithmetic sequence is a sequence in which successive terms differ by a constant amount. The common difference in 25.b. is 2; and the common difference in 25.c. is -4 .



28. a. To divide AB into four congruent parts, begin by determining the midpoint, C , of AB . Let

$$(x_1, y_1) = (2, 3) \text{ and } (x_2, y_2) = (6, 11).$$

$$\begin{aligned} C &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2+6}{2}, \frac{3+11}{2} \right) \\ &= \left(\frac{8}{2}, \frac{14}{2} \right) \\ &= (4, 7) \end{aligned}$$

Next, find D and E , the midpoints of AC and BC respectively.

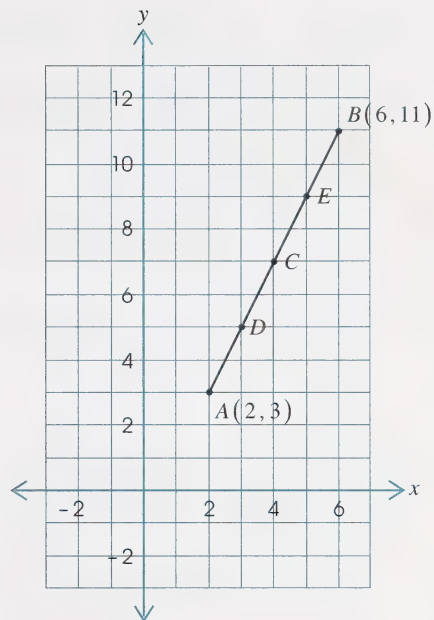
For D , let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 7)$.

$$\begin{aligned} \therefore D &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2+4}{2}, \frac{3+7}{2} \right) \\ &= \left(\frac{6}{2}, \frac{10}{2} \right) \\ &= (3, 5) \end{aligned}$$

For E , let $(x_1, y_1) = (6, 11)$ and $(x_2, y_2) = (4, 7)$.

$$\begin{aligned} \therefore E &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{6+4}{2}, \frac{11+7}{2} \right) \\ &= \left(\frac{10}{2}, \frac{18}{2} \right) \\ &= (5, 9) \end{aligned}$$

The points that divide AB into four congruent parts are $(3, 5)$, $(4, 7)$, and $(5, 9)$.



Section 1: Activity 2 (continued)

To divide XY into eight congruent parts, begin by determining the midpoint, M , of XY . Let

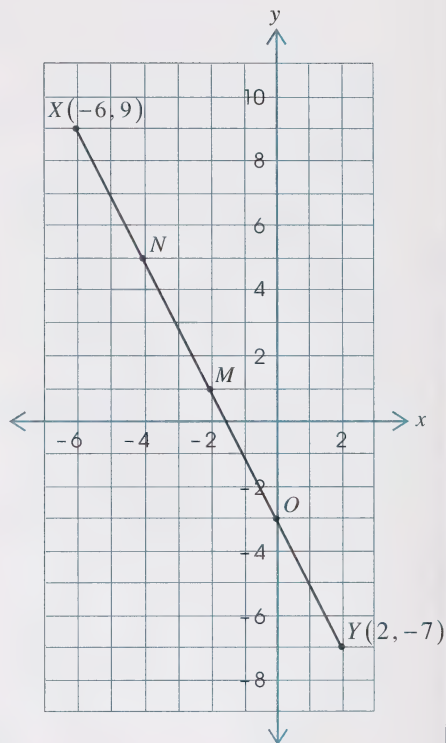
$$(x_1, y_1) = (-6, 9) \text{ and } (x_2, y_2) = (2, -7).$$

$$\begin{aligned} M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-6 + 2}{2}, \frac{9 + (-7)}{2} \right) \\ &= \left(\frac{-4}{2}, \frac{2}{2} \right) \\ &= (-2, 1) \end{aligned}$$

Next, find N and O , the midpoints of MX and MY respectively.

$$\begin{aligned} \text{For } N, \text{ let } (x_1, y_1) &= (-6, 9) \text{ and} \\ (x_2, y_2) &= (-2, 1). \end{aligned}$$

$$\begin{aligned} \therefore N &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-6 + (-2)}{2}, \frac{9 + 1}{2} \right) \\ &= \left(\frac{-8}{2}, \frac{10}{2} \right) \\ &= (-4, 5) \end{aligned}$$



For O , let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (2, -7)$.

$$\begin{aligned}\therefore O &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 2}{2}, \frac{1 + (-7)}{2} \right) \\ &= \left(\frac{0}{2}, \frac{-6}{2} \right) \\ &= (0, -3)\end{aligned}$$

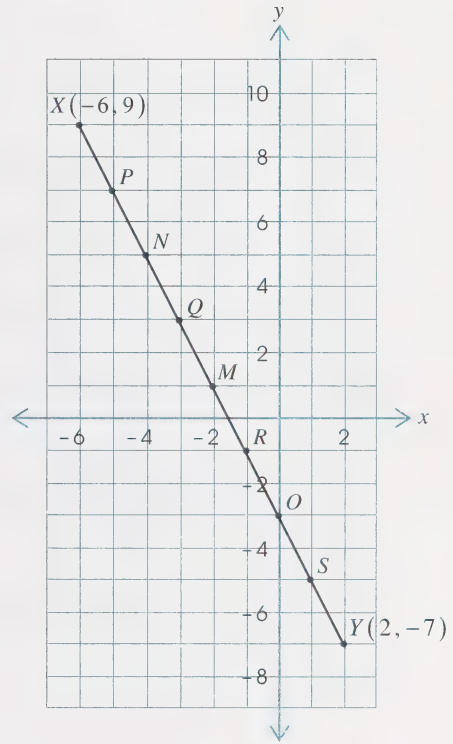
Finally, find P , Q , R , and S , the midpoints of NX , MN , MO , and OY respectively.

For P , let $(x_1, y_1) = (-6, 9)$
and $(x_2, y_2) = (-4, 5)$.

$$\begin{aligned}\therefore P &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-6 + (-4)}{2}, \frac{9 + 5}{2} \right) \\ &= \left(\frac{-10}{2}, \frac{14}{2} \right) \\ &= (-5, 7)\end{aligned}$$

For Q , let $(x_1, y_1) = (-4, 5)$
and $(x_2, y_2) = (-2, 1)$.

$$\begin{aligned}\therefore Q &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-4 + (-2)}{2}, \frac{5 + 1}{2} \right) \\ &= \left(\frac{-6}{2}, \frac{6}{2} \right) \\ &= (-3, 3)\end{aligned}$$



Section 1: Activity 2 (continued)

For R , let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (0, -3)$.

$$\begin{aligned}\therefore R &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 0}{2}, \frac{1 + (-3)}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{-2}{2} \right) \\ &= (-1, -1)\end{aligned}$$

For S , let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (2, -7)$.

$$\begin{aligned}\therefore S &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{0 + 2}{2}, \frac{-3 + (-7)}{2} \right) \\ &= \left(\frac{2}{2}, \frac{-10}{2} \right) \\ &= (1, -5)\end{aligned}$$

The points that divide XY into eight congruent parts are $(-5, 7)$, $(-4, 5)$, $(-3, 3)$, $(-2, 1)$, $(-1, -1)$, $(0, -3)$, and $(1, -5)$.

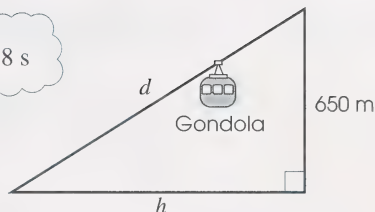
- b. No, you can't use the midpoint formula to divide a line segment into three congruent parts. You can use the midpoint formula to obtain two equal parts, four equal parts, eight equal parts, and so on.

31. a. Distance = speed $\times$ time

$$\begin{aligned} d &= 5.09 \text{ m/s} \times 403.8 \text{ s} \\ &\doteq 2060 \text{ m} \end{aligned}$$

$$6.73 \text{ min} = 403.8 \text{ s}$$

The distance travelled by the lift is approximately 2060 m.



- b. Use the Pythagorean Theorem to find the horizontal distance.

$$h^2 = d^2 - 650^2$$

$$h^2 \doteq 2060^2 - 650^2$$

$$\begin{aligned} h &\doteq \sqrt{2060^2 - 650^2} \\ &\doteq 1950 \end{aligned}$$

The horizontal distance travelled by the gondola is approximately 1950 m.

- c. The 25 support towers divide the distance, d , into 26 congruent segments.

The horizontal distance between each support tower is about $\frac{1950}{26} = 75 \text{ m}$.

The vertical distance between each support tower is $\frac{650}{26} = 25 \text{ m}$.

If the lower terminal has coordinates $(0, 0)$, the upper terminal will have coordinates $(1950, 650)$.

The coordinates of the cable at the tenth tower below the upper terminal are

$$\begin{aligned} (1950 - 10(75), 650 - 10(25)) &= (1950 - 750, 650 - 250) \\ &= (1200, 400) \end{aligned}$$

This solution assumes that the cable lies along a straight line and that the support towers are evenly spaced.

Section 1: Activity 2 (continued)

35. b. If C divides AB into the ratio of 2:3, you can think of C as the second division point from A when AB is divided into $2 + 3 = 5$ congruent parts.

The horizontal distance from A to B is

$$\begin{aligned}|x_2 - x_1| &= |13 - (-2)| \\ &= 15\end{aligned}$$

The horizontal distance between each division point is $\frac{15}{5}$ or 3.

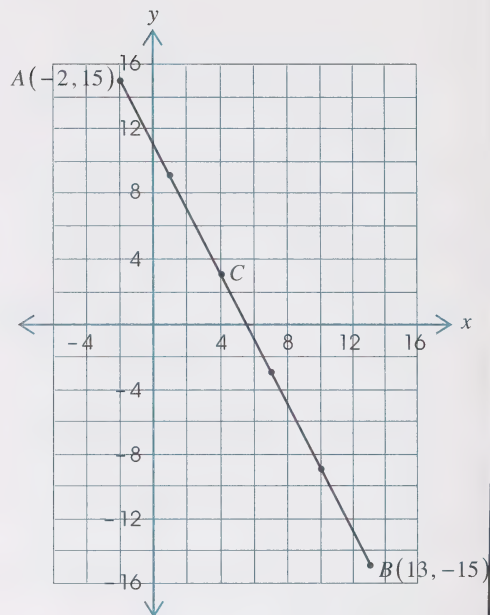
The vertical distance from A to B is

$$\begin{aligned}|y_2 - y_1| &= |(-15) - 15| \\ &= 30\end{aligned}$$

The vertical distance between each division point is $\frac{30}{5}$ or 6.

Point C is the second division point down and to the right of A .

The coordinates of C are $(-2 + 2(3), 15 - 2(6))$ or $(4, 3)$.



3. Textbook questions 1 to 8 of “Mental Math: Order of Operations,” p. 453

$$\begin{aligned}1. \text{ Mean} &= \frac{5+11}{2} \\ &= 8\end{aligned}$$

$$\begin{aligned}2. \text{ Mean} &= \frac{4+(-6)}{2} \\ &= -1\end{aligned}$$

$$\begin{aligned}3. \text{ Mean} &= \frac{-2+8}{2} \\ &= 3\end{aligned}$$

$$\begin{aligned}4. \text{ Mean} &= \frac{-3+(-7)}{2} \\ &= -5\end{aligned}$$

$$\begin{aligned}5. \text{ Mean} &= \frac{7+4}{2} \\ &= 5.5\end{aligned}$$

$$\begin{aligned}6. \text{ Mean} &= \frac{0+(-9)}{2} \\ &= -4.5\end{aligned}$$

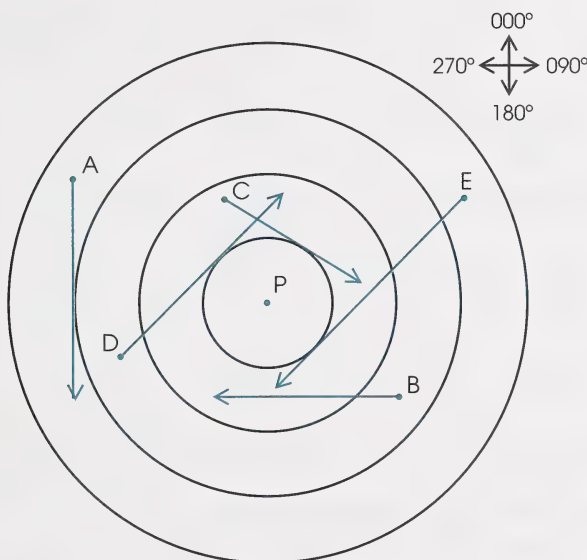
$$\begin{aligned}7. \text{ Mean} &= \frac{-1+(-6)}{2} \\ &= -3.5\end{aligned}$$

$$\begin{aligned}8. \text{ Mean} &= \frac{3+(-12)}{2} \\ &= -4.5\end{aligned}$$

Section 1: Activity 3

1. a. Textbook question a. of “Explore: Use the Diagram,” p. 470

- a. The scale is 1 cm = 5 km, or 1:500 000.



b. Textbook questions 1 to 4 of “Inquire,” p. 470

1. Answers may vary slightly.
 - a. Ship A, on a course of 180° , comes within 15 km of the platform.
 - b. Ship B, on a course of 270° , comes within 7.5 km of the platform.
 - c. Ship C, on a course of 120° , comes within 5 km of the platform.
2. The segment representing the closest distance from a ship to the platform is a perpendicular line from the platform to the course of the ship.
3.
 - a. Their courses are parallel.
 - b. Ships D and E are about 10 km apart, when they are closest together.
4. The line segment representing the closest distance between ships D and E is perpendicular to the lines representing the ships' courses.

Section 1: Activity 3 (continued)

2. a. Textbook questions 1, 9, 18, 23, 32, 35, 39, and 42 of “Practice,” p. 475

$$1. \quad y = 4x - 1$$

$$y = mx + b$$

$$\therefore m = 4$$

The slope of a perpendicular line is $-\frac{1}{4}$, because $4 \times \left(-\frac{1}{4}\right) = -1$.

9. Use the point-slope formula. Let $m = 1$ and $(x_1, y_1) = (1, 2)$.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = 1(x - 1)$$

$$y - 2 = x - 1$$

$$y = x + 1$$

The equation of the line that passes through $(1, 2)$ and has a slope of 1 is $y = x + 1$.

18. The shortest distance is the perpendicular distance, OA , from the point $O(0, 0)$ to the line $y = -2x - 5$.

To find the length of OA , first find the coordinates of A .

The slope of $y = -2x - 5$ is -2 .

Because $y = -2x - 5$ and OA are perpendicular, the product of their slopes is -1 .

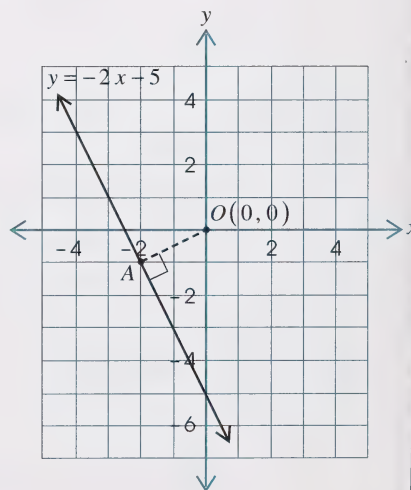
So, the slope of OA is $\frac{1}{2}$.

To find the equation for OA , use the point-slope formula. Let $m = \frac{1}{2}$ and $(x_1, y_1) = (0, 0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = \frac{1}{2}(x - 0)$$

$$y = \frac{1}{2}x$$



To find the coordinates of A , solve the following system of equations:

$$y = -2x - 5 \quad (1)$$

$$y = \frac{1}{2}x \quad (2)$$

Substitute $\frac{1}{2}x$ for y into (1).

$$\frac{1}{2}x = -2x - 5$$

$$2\frac{1}{2}x = -5$$

$$\frac{5}{2}x = -5$$

$$5x = -10$$

$$x = -2$$

Substitute $x = -2$ into (2).

$$y = \frac{1}{2}x$$

$$= \frac{1}{2}(-2)$$

$$= -1$$

The coordinates of A are $(-2, -1)$.

Use the distance formula to find the distance from O to A . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (-2, -1)$.

$$\begin{aligned} OA &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-2 - 0)^2 + (-1 - 0)^2} \\ &= \sqrt{(-2)^2 + (-1)^2} \\ &= \sqrt{4 + 1} \\ &= \sqrt{5} \end{aligned}$$

The shortest distance from the origin to the line $y = -2x - 5$ is $\sqrt{5}$.

Section 1: Activity 3 (continued)

23. The shortest distance is the perpendicular distance, OA , from the point $O(0, 0)$ to the line $x - y = 3$.

To find the length of OA , first find the coordinates of A .

Find the slope of $x - y = 3$.

$$x - y = 3$$

$$x - 3 = y$$

$$y = 1x - 3$$

$$y = mx + b$$

$$\therefore m = 1$$

Since $x - y = 3$ and OA are perpendicular, the product of their slopes is -1 .

So, the slope of OA is -1 .

To find the equation of OA , use the point-slope formula. Let $m = -1$ and $(x_1, y_1) = (0, 0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -1(x - 0)$$

$$y = -x$$

To find the coordinates of A , solve the following system of equations:

$$x - y = 3 \quad (1)$$

$$y = -x \quad (2)$$

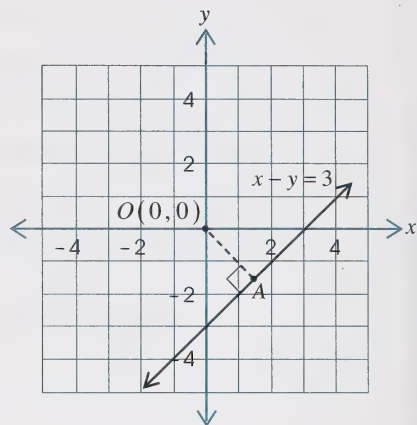
Substitute $-x$ for y into (1).

$$x - y = 3$$

$$x - (-x) = 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$



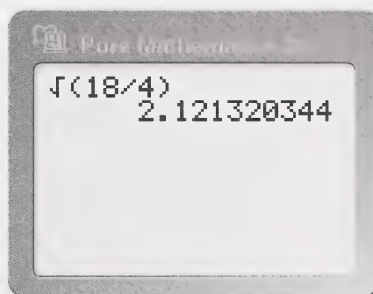
Substitute $x = \frac{3}{2}$ into (2).

$$\begin{aligned} y &= -x \\ &= -\frac{3}{2} \end{aligned}$$

The coordinates of A are $(\frac{3}{2}, -\frac{3}{2})$.

Use the distance formula to find the distance from O to A . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (\frac{3}{2}, -\frac{3}{2})$.

$$\begin{aligned} OA &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{3}{2} - 0\right)^2 + \left(-\frac{3}{2} - 0\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{9}{4}} \\ &= \sqrt{\frac{18}{4}} \\ &\approx 2.1 \end{aligned}$$

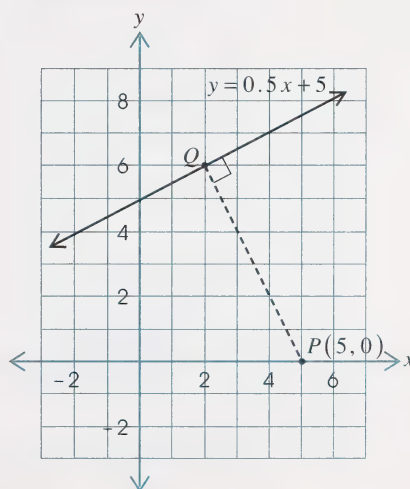


The shortest distance from the origin to the line $x - y = 3$ is approximately 2.1.

32. Find the coordinates of Q , the point of intersection of the line $y = 0.5x + 5$ and the perpendicular from $P(5, 0)$.

The line $y = 0.5x + 5$ has a slope of 0.5; therefore, the perpendicular PQ has a slope of -2 . Determine an equation of the line with slope -2 and passing through $P(5, 0)$. Use the point-slope formula.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -2(x - 5) \\ y &= -2x + 10 \end{aligned}$$



Section 1: Activity 3 (continued)

To find the coordinates of Q , solve the following system of equations:

$$y = 0.5x + 5 \quad (1)$$

$$y = -2x + 10 \quad (2)$$

Substitute $-2x + 10$ for y into (1).

$$y = 0.5x + 5$$

$$-2x + 10 = 0.5x + 5$$

$$-2.5x = -5$$

$$x = 2$$

Substitute $x = 2$ into (1).

$$y = 0.5x + 5$$

$$= 0.5(2) + 5$$

$$= 6$$

The coordinates of Q are $(2, 6)$.

Use the distance formula to find the distance from P to Q . Let $(x_1, y_1) = (5, 0)$ and $(x_2, y_2) = (2, 6)$.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 5)^2 + (6 - 0)^2} \\ &= \sqrt{(-3)^2 + 6^2} \\ &= \sqrt{9 + 36} \\ &= \sqrt{45} \\ &\doteq 6.7 \end{aligned}$$

The shortest distance from $(5, 0)$ to the line of $y = 0.5x + 5$ is approximately 6.7.

35. Find the coordinates of Q , the point of intersection of the line $2x - y + 3 = 0$ and the perpendicular from $P(3, -1)$.

The line $2x - y + 3 = 0$, or $y = 2x + 3$, has a slope of 2; therefore, the perpendicular PQ has a slope of $-\frac{1}{2}$.

Use the point-slope formula to find the equation of the line with a slope of $-\frac{1}{2}$ and passing through

$P(3, -1)$. Let $m = -\frac{1}{2}$ and $(x_1, y_1) = (3, -1)$.

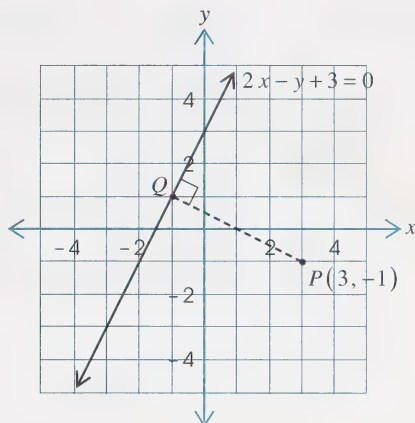
$$y - y_1 = m(x - x_1)$$

$$y - (-1) = -\frac{1}{2}(x - 3)$$

$$y + 1 = -\frac{1}{2}x + \frac{3}{2}$$

$$2y + 2 = -x + 3$$

$$x + 2y = 1$$



To find the coordinates of Q , solve the following system of equations:

$$2x - y = -3 \quad (1)$$

$$x + 2y = 1 \quad (2)$$

$$x + 2y = 1 \quad (2)$$

$$4x - 2y = -6 \quad (3): 2 \times (1)$$

$$\begin{array}{r} 4x - 2y = -6 \\ x + 2y = 1 \\ \hline 5x = -5 \end{array} \quad (2) + (3)$$

$$x = -1$$

Substitute $x = -1$ into (1).

$$2x - y = -3$$

$$2(-1) - y = -3$$

$$-2 - y = -3$$

$$-y = -1$$

$$y = 1$$

The coordinates of Q are $(-1, 1)$.

Section 1: Activity 3 (continued)

Use the distance formula to find the distance from P to Q . Let $(x_1, y_1) = (3, -1)$ and $(x_2, y_2) = (-1, 1)$.

$$\begin{aligned}
 PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-1 - 3)^2 + (1 - (-1))^2} \\
 &= \sqrt{(-4)^2 + 2^2} \\
 &= \sqrt{16 + 4} \\
 &= \sqrt{20} \\
 &\doteq 4.5
 \end{aligned}$$

The shortest distance from $P(3, -1)$ to the line $2x - y + 3 = 0$ is approximately 4.5.

39. Graph $y = x + 5$ and $y = x - 2$.

The vertical distance between the lines is the difference between the y -intercepts.

The y -intercept of $y = x + 5$ is 5; the y -intercept of $y = x - 2$ is -2 .

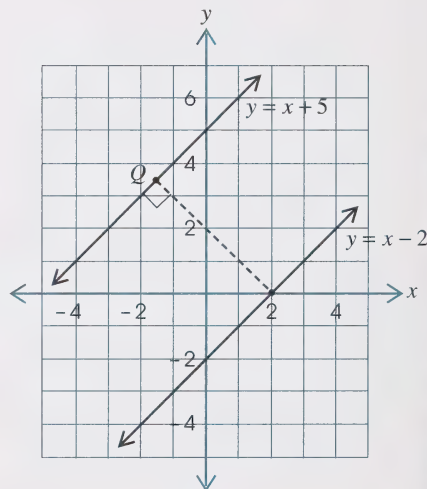
The difference between the y -intercepts is

$$\begin{aligned}
 |y_2 - y_1| &= |5 - (-2)| \\
 &= 7
 \end{aligned}$$

The vertical distance between the lines is 7.

The horizontal distance between the lines is the difference between the x -intercepts. To find the x -intercepts, let $y = 0$.

$$\begin{array}{ll}
 y = x + 5 & \text{and} \quad y = x - 2 \\
 0 = x + 5 & \quad 0 = x - 2 \\
 x = -5 & \quad x = 2
 \end{array}$$



The difference between the x -intercepts is

$$\begin{aligned} |x_2 - x_1| &= |-5 - 2| \\ &= |-7| \\ &= 7 \end{aligned}$$

The horizontal distance between the lines is 7.

To find the shortest distance between the two lines, choose a point on one line. For example, choose $P(2, 0)$ on $y = x - 2$.

Determine the distance $P(2, 0)$ lies from the line $y = x + 5$. Find PQ , the length of the perpendicular from $P(2, 0)$ to the other line.

Because the slope of $y = x + 5$ is 1, the slope of PQ is -1 .

Use the point-slope formula to determine an equation for PQ . Let $m = -1$ and

$$(x_1, y_1) = (2, 0).$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -1(x - 2) \\ y &= -x + 2 \end{aligned}$$

To find the coordinates of Q , solve the following system of equations:

$$\begin{aligned} y &= x + 5 & \textcircled{1} \\ y &= -x + 2 & \textcircled{2} \end{aligned}$$

Substitute $-x + 2$ for y into $\textcircled{1}$.

$$\begin{aligned} y &= x + 5 \\ -x + 2 &= x + 5 \\ -2x &= 3 \\ x &= -1.5 \end{aligned}$$

Section 1: Activity 3 (continued)

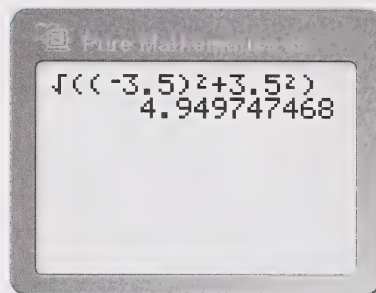
Substitute $x = -1.5$ into ①.

$$\begin{aligned} y &= x + 5 \\ &= -1.5 + 5 \\ &= 3.5 \end{aligned}$$

The coordinates of Q are $(-1.5, 3.5)$.

Use the distance formula to find the distance from P to Q . Let $(x_1, y_1) = (2, 0)$ and $(x_2, y_2) = (-1.5, 3.5)$.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-1.5 - 2)^2 + (3.5 - 0)^2} \\ &= \sqrt{(-3.5)^2 + 3.5^2} \\ &\doteq 4.95 \end{aligned}$$



The shortest distance between the lines is approximately 4.95.

42. Graph $3x + y = 5$ and $3x + y = 11$.

The vertical distance between the lines is the difference between the y -intercepts.

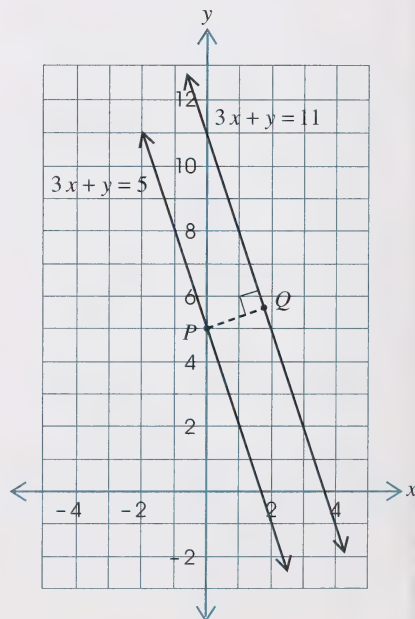
The y -intercept of $3x + y = 5$ (or $y = -3x + 5$) is 5;
the y -intercept of $3x + y = 11$ (or $y = -3x + 11$) is 11.

The difference between y -intercepts is

$$\begin{aligned} |y_2 - y_1| &= |5 - 11| \\ &= 6 \end{aligned}$$

The horizontal distance between the lines is the difference between the x -intercepts. To find the x -intercepts, let $y = 0$.

$$\begin{array}{ll} 3x + y = 5 & \text{and} \quad 3x + y = 11 \\ 3x + 0 = 5 & 3x + 0 = 11 \\ x = \frac{5}{3} & x = \frac{11}{3} \end{array}$$



The difference between x -intercepts is

$$\begin{aligned} |x_2 - x_1| &= \left| \frac{5}{3} - \frac{11}{3} \right| \\ &= \left| -\frac{6}{3} \right| \\ &= 2 \end{aligned}$$

The horizontal distance between the lines is 2.

To find the shortest distance between the lines, choose a point on one line. For example, choose $P(0, 5)$ on the line $3x + y = 5$. Determine the distance $P(0, 5)$ lies from the line $3x + y = 11$.

Find PQ , the length of the perpendicular from $P(0, 5)$ to the other line. Because the slope of $3x + y = 11$ (or $y = -3x + 11$) is -3 , the slope of PQ is $\frac{1}{3}$.

Use the point-slope formula to determine the equation of PQ . Let $m = \frac{1}{3}$ and $(x_1, y_1) = (0, 5)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= \frac{1}{3}(x - 0) \\ y - 5 &= \frac{1}{3}x \\ y &= \frac{1}{3}x + 5 \end{aligned}$$

To find the coordinates of Q , solve the following system of equations:

$$\begin{aligned} 3x + y &= 11 & \textcircled{1} \\ y &= \frac{1}{3}x + 5 & \textcircled{2} \end{aligned}$$

Substitute $\frac{1}{3}x + 5$ for y into $\textcircled{1}$.

$$\begin{aligned} 3x + \frac{1}{3}x + 5 &= 11 \\ 3\frac{1}{3}x &= 6 \\ \frac{10}{3}x &= 6 \\ 10x &= 18 \\ x &= 1.8 \end{aligned}$$

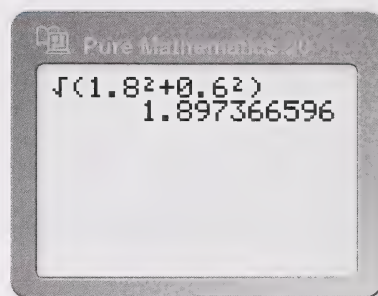
Section 1: Activity 3 (continued)

Substitute $x = 1.8$ into (2).

$$\begin{aligned} y &= \frac{1}{3}x + 5 \\ &= \frac{1}{3}(1.8) + 5 \\ &= 0.6 + 5 \\ &= 5.6 \end{aligned}$$

The coordinates of Q are $(1.8, 5.6)$. Use the distance formula to find PQ . Let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (1.8, 5.6)$.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(1.8 - 0)^2 + (5.6 - 5)^2} \\ &= \sqrt{1.8^2 + 0.6^2} \\ &\doteq 1.90 \end{aligned}$$



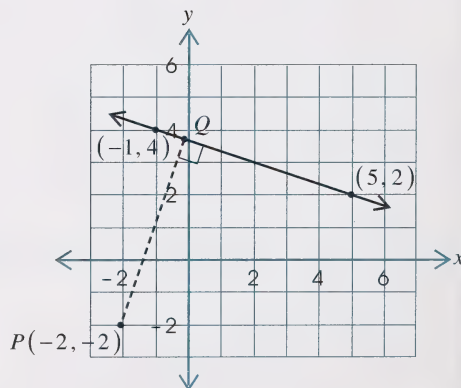
The shortest distance between the two lines is approximately 1.90.

b. Textbook questions 47, 48, 53, and 56 of “Applications and Problem Solving,” pp. 475 and 476

- 47.** Begin by finding the equation of the line passing through $(-1, 4)$ and $(5, 2)$.

First, find the slope of the line. Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (5, 2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 4}{5 - (-1)} \\ &= \frac{-2}{6} \\ &= -\frac{1}{3} \end{aligned}$$



Now, use the point-slope formula to determine the equation of the line. Let $m = \frac{1}{3}$ and

$$(x_1, y_1) = (-1, 4).$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{3}[x - (-1)]$$

$$y - 4 = -\frac{1}{3}(x + 1)$$

$$3y - 12 = -x - 1$$

$$x + 3y = 11$$

Because PQ is perpendicular to $x + 3y = 11$, the slope of PQ is 3.

Use the point-slope formula to determine the equation of PQ . Let $m = 3$ and

$$(x_1, y_1) = (-2, -2).$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = 3[x - (-2)]$$

$$y + 2 = 3(x + 2)$$

$$y + 2 = 3x + 6$$

$$0 = 3x - y + 4$$

$$3x - y = -4$$

Next, find the coordinates of Q by solving the following system of equations:

$$x + 3y = 11 \quad (1)$$

$$3x - y = -4 \quad (2)$$

$$\begin{array}{rcl} x + 3y & = & 11 \quad (1) \\ 9x - 3y & = & -12 \quad (3) : 3 \times (2) \\ \hline 10x & = & -1 \\ x & = & -0.1 \end{array}$$

Section 1: Activity 3 (continued)

Substitute $x = -0.1$ into (1).

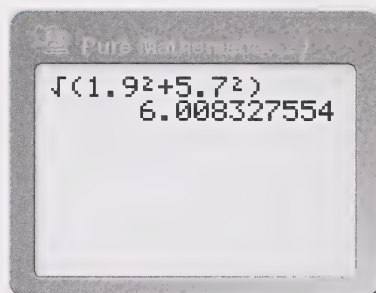
$$\begin{aligned}x + 3y &= 11 \\-0.1 + 3y &= 11 \\3y &= 11.1 \\y &= 3.7\end{aligned}$$

The coordinates of Q are $(-0.1, 3.7)$.

Use the distance formula to find the distance from P to Q .

Let $(x_1, y_1) = (-2, -2)$ and $(x_2, y_2) = (-0.1, 3.7)$.

$$\begin{aligned}PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{[-0.1 - (-2)]^2 + [3.7 - (-2)]^2} \\&= \sqrt{1.9^2 + 5.7^2} \\&\doteq 6.01\end{aligned}$$



The shortest distance from $(-2, -2)$ to the line through $(-1, 4)$ and $(5, 2)$ is approximately 6.01.

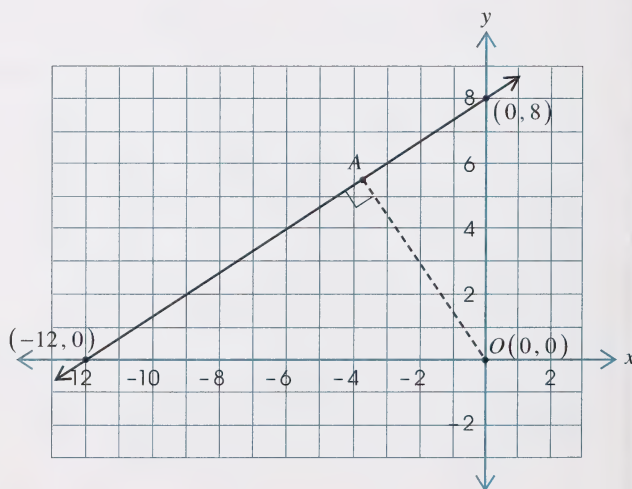
48. Begin by finding the equation of the line with a y -intercept of 8 and an x -intercept of -12 .

First, calculate the slope of this

line. Let $(x_1, y_1) = (-12, 0)$

and $(x_2, y_2) = (0, 8)$.

$$\begin{aligned}m &= \frac{y_2 - y_1}{x_2 - x_1} \\&= \frac{8 - 0}{0 - (-12)} \\&= \frac{8}{12} \\&= \frac{2}{3}\end{aligned}$$



Use the slope and y-intercept formula to determine the equation of this line. Let $m = \frac{2}{3}$ and $b = 8$.

$$y = mx + b$$

$$y = \frac{2}{3}x + 8$$

Because OA is perpendicular to $y = \frac{2}{3}x + 8$, the slope of OA is $-\frac{3}{2}$.

Use the point-slope formula to determine the equation of line OA . Let $m = -\frac{3}{2}$ and $(x_1, y_1) = (0, 0)$.

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{3}{2}(x - 0)$$

$$y = -\frac{3}{2}x$$

Now, find the coordinates of A by solving the following system of equations:

$$y = \frac{2}{3}x + 8 \quad (1)$$

$$y = -\frac{3}{2}x \quad (2)$$

Substitute $-\frac{3}{2}x$ for y into (1).

$$y = \frac{2}{3}x + 8$$

$$-\frac{3}{2}x = \frac{2}{3}x + 8$$

$$6\left(-\frac{3}{2}x\right) = 6\left(\frac{2}{3}x\right) + 6(8)$$

$$-9x = 4x + 48$$

$$-13x = 48$$

$$x = -\frac{48}{13}$$

Section 1: Activity 3 (continued)

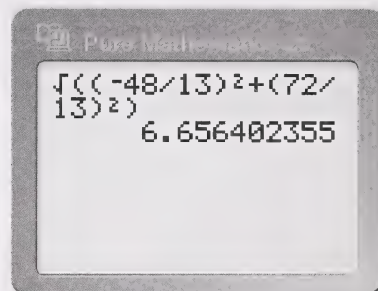
Substitute $x = -\frac{48}{13}$ into (2).

$$\begin{aligned} y &= -\frac{3}{2}x \\ &= -\frac{3}{2}\left(-\frac{48}{13}\right) \\ &= \frac{72}{13} \end{aligned}$$

The coordinates of A are $\left(-\frac{48}{13}, \frac{72}{13}\right)$.

Use the distance formula to find the distance from O to A . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = \left(-\frac{48}{13}, \frac{72}{13}\right)$.

$$\begin{aligned} OA &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(-\frac{48}{13} - 0\right)^2 + \left(\frac{72}{13} - 0\right)^2} \\ &= \sqrt{\left(-\frac{48}{13}\right)^2 + \left(\frac{72}{13}\right)^2} \\ &\doteq 6.7 \end{aligned}$$



The shortest distance from the origin to the line with a y -intercept of 8 and an x -intercept of -12 is approximately 6.7.

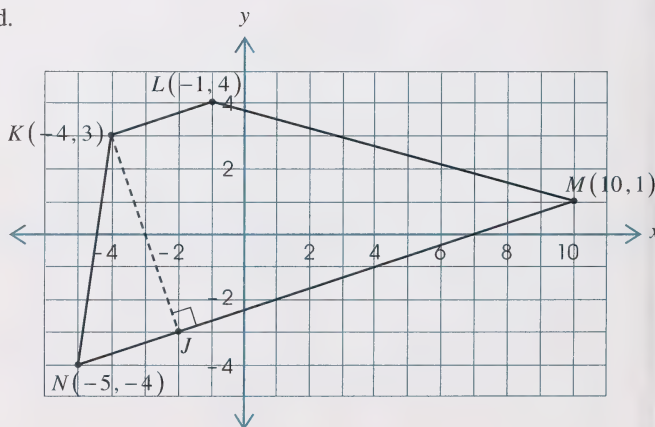
53. Verify that $KLMN$ is a trapezoid.

Find the slope of KL . Let

$(x_1, y_1) = (-4, 3)$ and

$(x_2, y_2) = (-1, 4)$.

$$\begin{aligned} m_{KL} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 3}{-1 - (-4)} \\ &= \frac{1}{3} \end{aligned}$$



Find the slope of MN . Let $(x_1, y_1) = (-5, -4)$ and $(x_2, y_2) = (10, 1)$.

$$\begin{aligned} m_{MN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - (-4)}{10 - (-5)} \\ &= \frac{5}{15} \\ &= \frac{1}{3} \end{aligned}$$

Because $m_{KL} = m_{MN}$, $KL \parallel MN$, thus making $KLMN$ a trapezoid.

To find the area of a trapezoid, determine the length of its parallel sides and the distance between them.

First, find the length of each parallel side.

For KL , let $(x_1, y_1) = (-4, 3)$ and $(x_2, y_2) = (-1, 4)$.

$$\begin{aligned} \therefore KL &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[-1 - (-4)]^2 + (4 - 3)^2} \\ &= \sqrt{3^2 + 1^2} \\ &= \sqrt{9 + 1} \\ &= \sqrt{10} \end{aligned}$$

For MN , let $(x_1, y_1) = (10, 1)$ and $(x_2, y_2) = (-5, -4)$.

$$\begin{aligned} \therefore MN &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-5 - 10)^2 + (-4 - 1)^2} \\ &= \sqrt{(-15)^2 + (-5)^2} \\ &= \sqrt{225 + 25} \\ &= \sqrt{250} \\ &= 5\sqrt{10} \end{aligned}$$

Section 1: Activity 3 (continued)

Now, find the altitude (the length of JK). Use the point-slope formula to determine the equation of the base, MN . Let $m = \frac{1}{3}$ and $(x_1, y_1) = (10, 1)$.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{3}(x - 10)$$

$$y - 1 = \frac{1}{3}x - \frac{10}{3}$$

$$y = \frac{1}{3}x - \frac{7}{3}$$

Determine the equation of the line passing through $K(-4, 3)$ that is perpendicular to $y = \frac{1}{3}x - \frac{7}{3}$ at J .

The slope of JK is -3 .

Use the point-slope formula to determine the equation of JK . Let $m = -3$ and $(x_1, y_1) = (-4, 3)$.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -3[x - (-4)]$$

$$y - 3 = -3(x + 4)$$

$$y - 3 = -3x - 12$$

$$y = -3x - 9$$

Determine the coordinates of J by solving the following system of equations:

$$y = \frac{1}{3}x - \frac{7}{3} \quad (1)$$

$$y = -3x - 9 \quad (2)$$

Substitute $-3x - 9$ for y into (1).

$$\begin{aligned}
 y &= \frac{1}{3}x - \frac{7}{3} \\
 -3x - 9 &= \frac{1}{3}x - \frac{7}{3} \\
 3(-3x - 9) &= 3\left(\frac{1}{3}x - \frac{7}{3}\right) \\
 -9x - 27 &= x - 7 \\
 -10x &= 20 \\
 x &= -2
 \end{aligned}$$

Substitute $x = -2$ into (2).

$$\begin{aligned}
 y &= -3x - 9 \\
 &= -3(-2) - 9 \\
 &= 6 - 9 \\
 &= -3
 \end{aligned}$$

The coordinates of J are $(-2, -3)$.

Use the distance formula to find the distance from J to K . Let $(x_1, y_1) = (-2, -3)$ and $(x_2, y_2) = (-4, 3)$.

$$\begin{aligned}
 JK &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[-4 - (-2)]^2 + [3 - (-3)]^2} \\
 &= \sqrt{(-2)^2 + 6^2} \\
 &= \sqrt{4 + 36} \\
 &= \sqrt{40} \\
 &= 2\sqrt{10}
 \end{aligned}$$

The altitude of the trapezoid is $2\sqrt{10}$.

Section 1: Activity 3 (continued)

Lastly, find the area of the trapezoid.

$$\begin{aligned}
 \text{Area} &= \frac{1}{2}(a+b)h \\
 &= \frac{1}{2}(\sqrt{10} + 5\sqrt{10})(2\sqrt{10}) \\
 &= \frac{1}{2}(6\sqrt{10})(2\sqrt{10}) \\
 &= 60
 \end{aligned}$$

$$a = \sqrt{10}, \quad b = 5\sqrt{10}, \quad \text{and} \quad h = 2\sqrt{10}$$

The area of the trapezoid is 60.

56. a. The vertical distance between the parallel lines is the difference between the y-intercepts.

The y-intercept of $y = 0.8x + 4.4$ is 4.4; the y-intercept of $y = 0.8x + 2.5$ is 2.5.

$$\begin{aligned}
 \therefore |y_2 - y_1| &= |4.4 - 2.5| \\
 &= 1.9
 \end{aligned}$$

Therefore, the tallest person that can walk down the tunnel is 1.9 m tall.

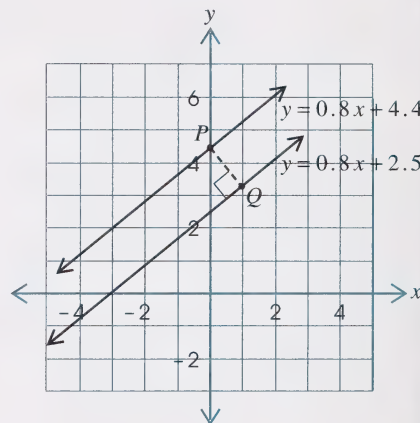
- b. The horizontal distance between the parallel lines is the difference between the x-intercepts.

To find the x-intercepts, let $y = 0$.

$$\begin{array}{ll}
 y = 0.8x + 4.4 & \text{and} \quad y = 0.8x + 2.5 \\
 0 = 0.8x + 4.4 & 0 = 0.8x + 2.5 \\
 -0.8x = 4.4 & -0.8x = 2.5 \\
 x = -5.5 & x = -3.125
 \end{array}$$

$$\begin{aligned}
 \therefore |x_2 - x_1| &= |-5.5 - (-3.125)| \\
 &= 2.375
 \end{aligned}$$

Therefore, if the tunnel were flooded, the distance across the water surface would be 2.375 m.



- c. The maximum height of a fork-lift would be the distance between the horizontal lines. Pick a point on $y = 0.8x + 4.4$. For example, choose $P(0, 4.4)$.

Determine the equation of the line PQ , the perpendicular distance between the lines.

Because the slope of $y = 0.8x + 2.5$ is 0.8, the slope of PQ is $-\frac{1}{0.8}$ or -1.25 .

Use the point-slope formula to determine the equation of PQ . Let $m = -1.25$ and $(x_1, y_1) = (0, 4.4)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 4.4 &= -1.25(x - 0) \\ y &= -1.25x + 4.4 \end{aligned}$$

Locate point Q by solving the following system of equations:

$$\begin{aligned} y &= 0.8x + 2.5 & \textcircled{1} \\ y &= -1.25x + 4.4 & \textcircled{2} \end{aligned}$$

Substitute $-1.25x + 4.4$ for y into $\textcircled{1}$.

$$\begin{aligned} y &= -1.25x + 4.4 \\ -1.25x + 4.4 &= 0.8x + 2.5 \\ -2.05x &= -1.9 \\ x &= \frac{-1.90}{-2.05} \\ &= \frac{190}{205} \\ &= \frac{38}{41} \end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $x = \frac{38}{41}$ into (1).

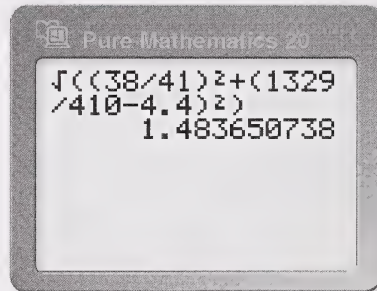
$$\begin{aligned}
 y &= 0.8x + 2.5 \\
 &= 0.8\left(\frac{38}{41}\right) + 2.5 \\
 &= \frac{30.4}{41} + \frac{102.5}{41} \\
 &= \frac{132.9}{41} \\
 &= \frac{1329}{410}
 \end{aligned}$$

The coordinates of Q are $\left(\frac{38}{41}, \frac{1329}{410}\right)$.

Use the distance formula to find the distance from P to Q . Let $(x_1, y_1) = (0, 4.4)$ and

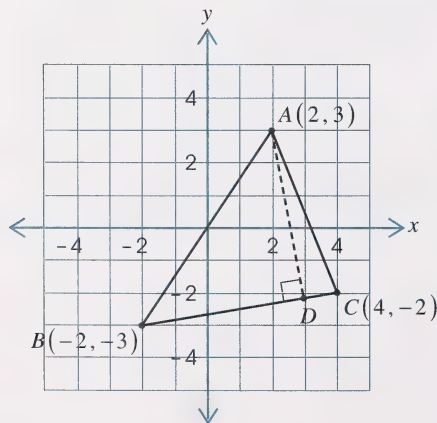
$$(x_2, y_2) = \left(\frac{38}{41}, \frac{1329}{410}\right).$$

$$\begin{aligned}
 PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(\frac{38}{41} - 0\right)^2 + \left(\frac{1329}{410} - 4.4\right)^2} \\
 &\doteq 1.5
 \end{aligned}$$



Because the shortest distance between the sides is approximately 1.5 m, a fork-lift 1.75 m tall could not be driven into the tunnel.

3. a.



To find the length of the altitude from A , first determine the equation of BC . Let $(x_1, y_1) = (-2, -3)$ and $(x_2, y_2) = (4, -2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - (-3)}{4 - (-2)} \\ &= \frac{1}{6} \end{aligned}$$

Use the point-slope formula to determine the equation of BC . Let $m = \frac{1}{6}$ and $(x_1, y_1) = (-2, -3)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= \frac{1}{6}[x - (-2)] \\ y + 3 &= \frac{1}{6}(x + 2) \\ y + 3 &= \frac{1}{6}x + \frac{1}{3} \\ y &= \frac{1}{6}x - \frac{8}{3} \end{aligned}$$

Now, determine the equation of the line through A that intersects BC at a right angle at point D .

The slope of AD is -6 .

Use the point-slope formula to determine the equation of AD . Let $m = -6$ and $(x_1, y_1) = (2, 3)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 3 &= -6(x - 2) \\ y - 3 &= -6x + 12 \\ y &= -6x + 15 \end{aligned}$$

Determine the coordinates of D by solving the following system of equations:

$$\begin{aligned} y &= \frac{1}{6}x - \frac{8}{3} & \textcircled{1} \\ y &= -6x + 15 & \textcircled{2} \end{aligned}$$

Section 1: Activity 3 (continued)

Substitute $-6x + 15$ for y into (1).

$$\begin{aligned}
 y &= \frac{1}{6}x - \frac{8}{3} \\
 -6x + 15 &= \frac{1}{6}x - \frac{8}{3} \\
 6(-6x + 15) &= 6\left(\frac{1}{6}x - \frac{8}{3}\right) \\
 -36x + 90 &= x - 16 \\
 -37x &= -106 \\
 x &= \frac{-106}{-37} \\
 &= \frac{106}{37}
 \end{aligned}$$

Substitute $x = \frac{106}{37}$ into (2).

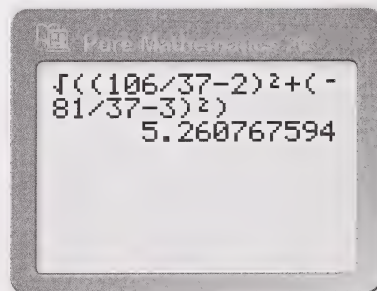
$$\begin{aligned}
 y &= -6x + 15 \\
 &= -6\left(\frac{106}{37}\right) + 15 \\
 &= -\frac{636}{37} + \frac{555}{37} \\
 &= -\frac{81}{37}
 \end{aligned}$$

The coordinates of D are $\left(\frac{106}{37}, -\frac{81}{37}\right)$.

Use the distance formula to find AD . Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = \left(\frac{106}{37}, -\frac{81}{37}\right)$.

$$\begin{aligned}
 AD &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(\frac{106}{37} - 2\right)^2 + \left(-\frac{81}{37} - 3\right)^2} \\
 &\doteq 5.26
 \end{aligned}$$

The altitude is approximately 5.26.



- b. To find the area of $\triangle ABC$, you need the lengths of the base and the altitude.

Use the distance formula to find the base, BC . Let $(x_1, y_1) = (-2, -3)$ and $(x_2, y_2) = (4, -2)$.

$$\begin{aligned} BC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[4 - (-2)]^2 + [-2 - (-3)]^2} \\ &= \sqrt{6^2 + 1^2} \\ &= \sqrt{37} \end{aligned}$$

$$\begin{aligned} \therefore \text{Area of } \triangle ABC &= \frac{1}{2} \text{base} \times \text{altitude} \\ &= \frac{1}{2} \sqrt{37} \times 5.26 \\ &\doteq 16.00 \end{aligned}$$

The area is approximately 16.00.

Section 1: Activity 4

1. a. Textbook questions a. to c. of “Explore: Draw a Graph,” p. 478

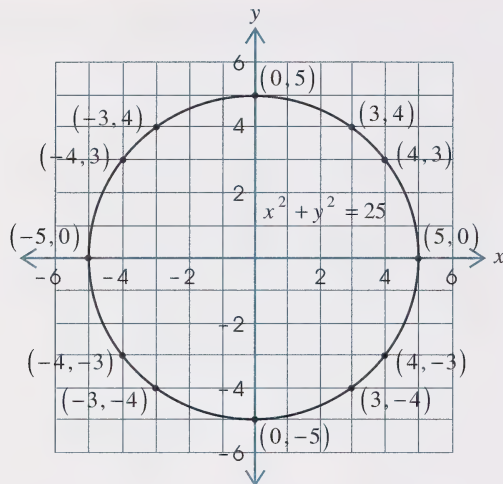
a.

x	5	4	3	0	-3	-4	-5	-4	-3	0	3	4
y	0	3	4	5	4	3	0	-3	-4	-5	-4	-3

- b. If $y \neq 0$, $x^2 + y^2 = 25$ or $y^2 = 25 - x^2$ is equivalent to $y = \pm \sqrt{25 - x^2}$.

Section 1: Activity 4 (continued)

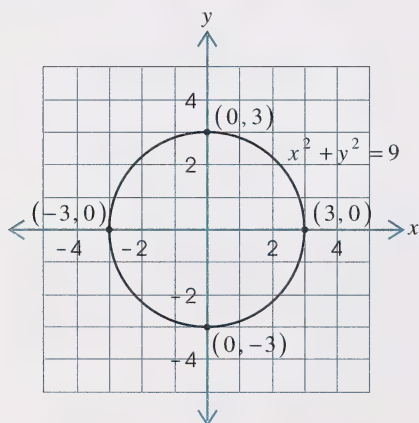
c.



b. Textbook questions 1 to 7 of “Inquire,” p. 478

1. The relation $x^2 + y^2 = 25$ is not a function because the values of y are not uniquely determined. When $y \neq 0$, there are two values of y for each value of x .
2. The centre of the graph is at the origin, $(0, 0)$.
3. The radius of the graph is 5.

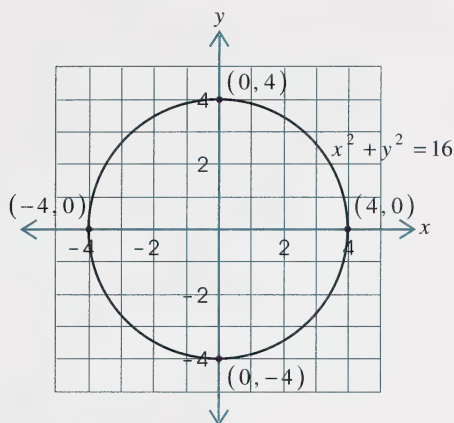
4. a.



- b. The centre is $(0, 0)$, and the radius is 3.

5. a. The centre of $x^2 + y^2 = 16$ is $(0, 0)$, and the radius is 4.

b.



6. The centre of $x^2 + y^2 = r^2$ is $(0, 0)$, and the radius is r .

7. a. The origin is at the centre of the planet Saturn.

b. $\sqrt{4489} = 67$

The radius of the innermost ring is 67×10^6 m or 67 000 km.

2. a. Textbook questions 1, 12, 13, 15, 22, 27, 29, and 35 of "Practice," p. 481

1. Use the general form of the equation of a circle.

$$(h, k) = (0, 0) \text{ and } r = 5$$

$$\therefore (x - h)^2 + (y - k)^2 = r^2$$

$$(x - 0)^2 + (y - 0)^2 = 5^2$$

$$x^2 + y^2 = 25$$

Section 1: Activity 4 (continued)

12. Use the general form of the equation of a circle.

$$(h, k) = (-3, -8) \text{ and } r = 10$$

$$\therefore (x-h)^2 + (y-k)^2 = r^2$$

$$[x - (-3)]^2 + [y - (-8)]^2 = 10^2$$

$$(x+3)^2 + (y+8)^2 = 100$$

13. Use the general form of the equation of a circle.

$$(h, k) = (-4, 0) \text{ and } r = 4\sqrt{2}$$

$$\therefore (x-h)^2 + (y-k)^2 = r^2$$

$$[x - (-4)]^2 + (y-0)^2 = (4\sqrt{2})^2$$

$$(x+4)^2 + y^2 = 16(2)$$

$$(x+4)^2 + y^2 = 32$$

15. Use the general form of the equation of a circle.

$$(h, k) = (0, 0)$$

$$\therefore (x-h)^2 + (y-k)^2 = r^2$$

$$(x-0)^2 + (y-0)^2 = r^2$$

$$x^2 + y^2 = r^2$$

If the circle passes through $(6, 8)$, the coordinates of this point satisfy the equation.

$$\therefore x^2 + y^2 = r^2$$

$$6^2 + 8^2 = r^2$$

$$36 + 64 = r^2$$

$$r^2 = 100$$

Therefore, the equation of the circle is $x^2 + y^2 = 100$.

22. Use the general form of the equation of a circle.

$$(h, k) = (-1, -2)$$

$$\begin{aligned}\therefore (x-h)^2 + (y-k)^2 &= r^2 \\ [x-(-1)]^2 + [y-(-2)]^2 &= r^2 \\ (x+1)^2 + (y+2)^2 &= r^2\end{aligned}$$

If the circle passes through $(3, 2)$, the coordinates of this point satisfy the equation.

$$\begin{aligned}\therefore (x+1)^2 + (y+2)^2 &= r^2 \\ (3+1)^2 + (2+2)^2 &= r^2 \\ 4^2 + 4^2 &= r^2 \\ 16 + 16 &= r^2 \\ r^2 &= 32\end{aligned}$$

Therefore, the equation of the circle is $(x+1)^2 + (y+2)^2 = 32$.

27. Compare $x^2 + y^2 = 49$ to $(x-h)^2 + (y-k)^2 = r^2$.

The centre, (h, k) , is $(0, 0)$, and the radius, r , is $\sqrt{49} = 7$.

29. Compare $(x+5)^2 + y^2 = 1$ to $(x-h)^2 + (y-k)^2 = r^2$.

The centre, (h, k) , is $(-5, 0)$, and the radius, r , is $\sqrt{1} = 1$.

35. Rewrite the equation to its general form, and compare it to $(x-h)^2 + (y-k)^2 = r^2$.

$$\begin{aligned}2(x-2)^2 + 2(y+1)^2 &= 32 \\ 2[(x-2)^2 + (y+1)^2] &= 32 \\ (x-2)^2 + (y+1)^2 &= 16\end{aligned}$$

Therefore, the centre, (h, k) , is $(2, -1)$, and the radius, r , is $\sqrt{16} = 4$.

Section 1: Activity 4 (continued)

- b. Textbook questions 37.b., 40.a., 42, 43, 45, and 49 of “Applications and Problem Solving,” pp. 481 and 482

37. b. If $(-4, 8)$ lies on the circle, it will satisfy the equation $(x+4)^2 + (y-3)^2 = 25$.

LS	RS
$(x+4)^2 + (y-3)^2$	25
$= (-4+4)^2 + (8-3)^2$	
$= 0^2 + 5^2$	
$= 25$	
LS	RS

Therefore, $(-4, 8)$ lies on the circle.

40. a. Find the centre of the circle by calculating the midpoint of diameter AB . Let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (2, 5)$.

$$\begin{aligned}
 \text{Centre} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{2+2}{2}, \frac{-5+5}{2} \right) \\
 &= \left(\frac{4}{2}, \frac{0}{2} \right) \\
 &= (2, 0)
 \end{aligned}$$

Use the general form of the equation of the circle.

$$(h, k) = (2, 0)$$

$$\therefore (x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-0)^2 = r^2$$

$$(x-2)^2 + y^2 = r^2$$

Because $A(2, -5)$ lies on the circle, its coordinates satisfy the equation.

$$\begin{aligned}\therefore (x-2)^2 + y^2 &= r^2 \\ (2-2)^2 + (-5)^2 &= r^2 \\ 0^2 + (-5)^2 &= r^2 \\ r^2 &= 25\end{aligned}$$

Therefore, the equation of the circle is $(x-2)^2 + y^2 = 25$.

- 42.** The radius of Earth's solid core is about $\frac{2400}{2}$ or 1200 km.

The equation that models a cross section of the core is

$$\begin{aligned}x^2 + y^2 &= 1200^2 \\ x^2 + y^2 &= 1\,440\,000\end{aligned}$$

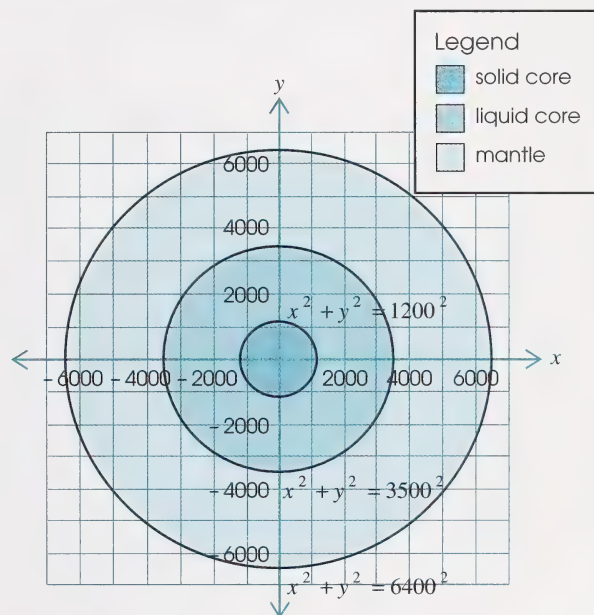
The liquid outer core extends from 1200 km to $1200 + 2300 = 3500$ km, measured from the centre of Earth.

The equation that models the outer circle of a cross section of the liquid outer core is

$$\begin{aligned}x^2 + y^2 &= 3500^2 \\ x^2 + y^2 &= 12\,250\,000\end{aligned}$$

The outermost layer, the mantle, extends from 3500 km to $3500 + 2900 = 6400$ km, measured from the centre of Earth. The equation that models the outer circle of a cross section of the mantle is

$$\begin{aligned}x^2 + y^2 &= 6400^2 \\ x^2 + y^2 &= 40\,960\,000\end{aligned}$$



Section 1: Activity 4 (continued)

43. The centre of Earth is $(0, 0)$.

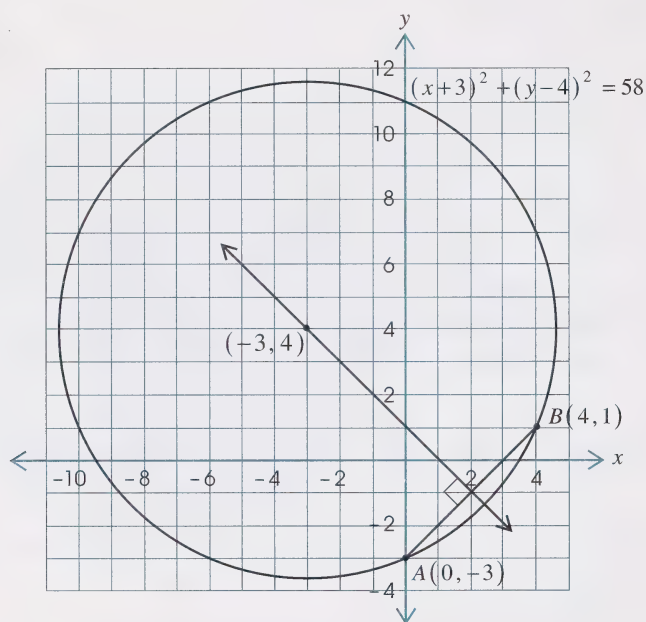
The radius of the orbit is $6370 + 35\,880 = 42\,250$ km.

The equation of the circular path of a geostationary satellite is

$$x^2 + y^2 = 42\,250^2$$

$$x^2 + y^2 = 1\,785\,062\,500$$

- 45.



- a. If the line segment joining $A(0, -3)$ and $B(4, 1)$ is a chord of the circle, then the coordinates of these points must satisfy the equation.

CheckFor $A(0, -3)$,

LS	RS
$(x+3)^2 + (y-4)^2$	58
$= (0+3)^2 + (-3-4)^2$	
$= 3^2 + (-7)^2$	
$= 9 + 49$	
$= 58$	
LS	RS

For $B(4, 1)$,

LS	RS
$(x+3)^2 + (y-4)^2$	58
$= (4+3)^2 + (1-4)^2$	
$= 7^2 + (-3)^2$	
$= 49 + 9$	
$= 58$	
LS	RS

Points $A(0, -3)$ and $B(4, 1)$ both lie on the circle.Therefore, AB is a chord of the circle.

- b. Determine the midpoint and slope of AB . Let $(x_1, y_1) = (0, -3)$ and $(x_2, y_2) = (4, 1)$.

$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{0+4}{2}, \frac{-3+1}{2} \right) \\ &= \left(\frac{4}{2}, \frac{-2}{2} \right) \\ &= (2, -1) \end{aligned}$	$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - (-3)}{4 - 0} \\ &= \frac{4}{4} \\ &= 1 \end{aligned}$
--	---

Because the slope of AB is 1, the slope of the perpendicular bisector of AB is -1 .Use the point-slope formula to determine the equation of the perpendicular bisector. Let $m = -1$ and $(x_1, y_1) = (2, -1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-1) &= -1(x - 2) \\ y + 1 &= -x + 2 \\ x + y - 1 &= 0 \end{aligned}$$

The equation of the perpendicular bisector of AB is $x + y - 1 = 0$.

Section 1: Activity 4 (continued)

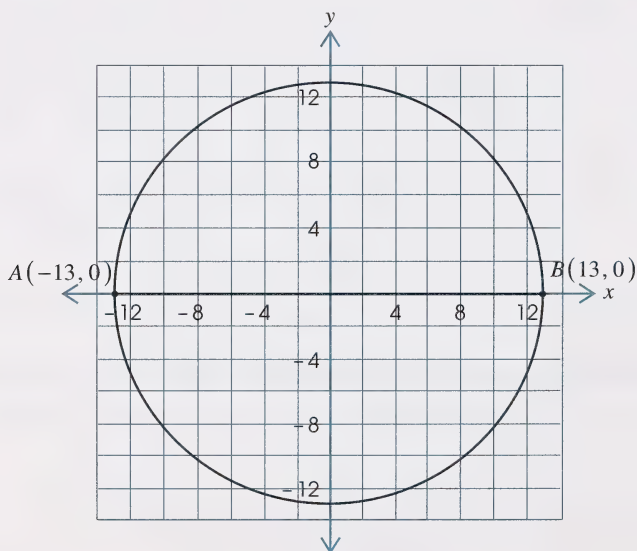
- c. Compare $(x+3)^2 + (y-4)^2 = 58$ to $(x-h)^2 + (y-k)^2 = r^2$. The centre, (h, k) , is $(-3, 4)$. If the centre $(-3, 4)$ of the circle lies on the perpendicular bisector of the chord AB , then the coordinates of the centre will satisfy the equation of the perpendicular bisector, $x + y - 1 = 0$.

LS	RS
$x + y - 1$	0
$= -3 + 4 - 1$	
$= 0$	
LS	= RS

Therefore, the centre of the circle lies on the perpendicular bisector of the chord AB .

49. a. The radius of the circle $x^2 + y^2 = 169$ is $\sqrt{169} = 13$. Therefore, the endpoints of the horizontal diameter are $A(-13, 0)$ and $B(13, 0)$.

Note: These endpoints are also the x-intercepts.



- b. If $P(-5, 12)$ lies on the circle, then the coordinates will satisfy the equation.

LS	RS
$x^2 + y^2$	169
$= (-5)^2 + 12^2$	
$= 25 + 144$	
$= 169$	
LS	= RS

Therefore, $P(-5, 12)$ lies on the circle.

- c. Find the slope of AP . Let

$$(x_1, y_1) = (-13, 0) \text{ and}$$

$$(x_2, y_2) = (-5, 12).$$

$$\begin{aligned}
 m_{AP} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{12 - 0}{-5 - (-13)} \\
 &= \frac{12}{8} \\
 &= \frac{3}{2}
 \end{aligned}$$

Find the slope of BP . Let

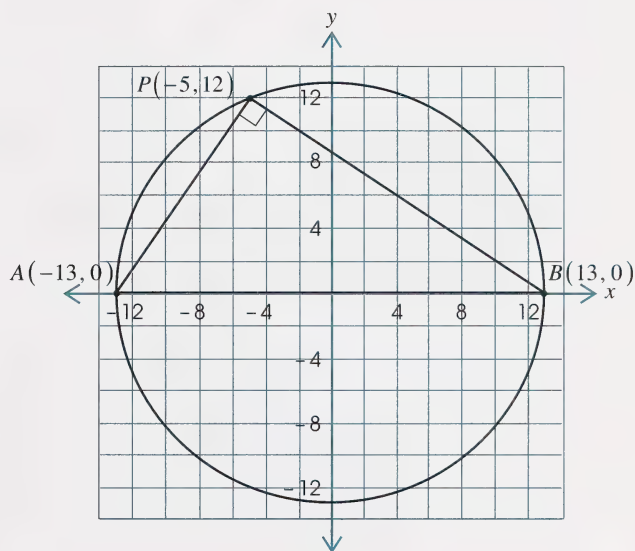
$$(x_1, y_1) = (13, 0) \text{ and}$$

$$(x_2, y_2) = (-5, 12).$$

$$\begin{aligned}
 m_{BP} &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{12 - 0}{-5 - 13} \\
 &= \frac{12}{-18} \\
 &= -\frac{2}{3}
 \end{aligned}$$

$$\text{Because } m_{AP} \times m_{BP} = \frac{3}{2} \left(-\frac{2}{3} \right) = -1, AP \perp BP.$$

Therefore, $\angle APB$ is a right angle.

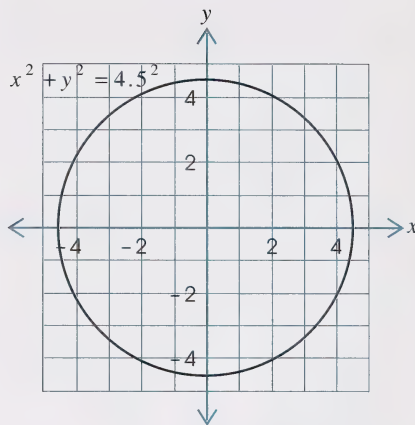


Section 1: Activity 4 (continued)

3. a. Textbook questions a. and b. of “Explore: Draw a Graph,” p. 483

a. $x^2 + y^2 = 4.5^2$ or $x^2 + y^2 = 20.25$

b.



b. Textbook questions 1 to 3 of “Inquire,” p. 483

1. a. Solve the system of equations.

$$x^2 + y^2 = 4.5^2 \quad (1)$$

$$y = x \quad (2)$$

Substitute x for y into (1).

$$x^2 + y^2 = 4.5^2$$

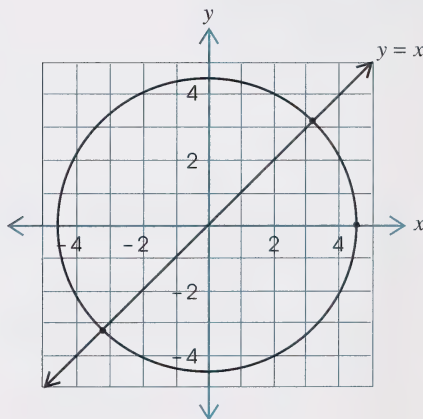
$$x^2 + x^2 = 4.5^2$$

$$2x^2 = 4.5^2$$

$$x^2 = \frac{4.5^2}{2}$$

$$x = \pm \sqrt{\frac{4.5^2}{2}}$$

$$\doteq \pm 3.2$$



When $x \doteq 3.2$, $y \doteq 3.2$. When $x \doteq -3.2$, $y \doteq -3.2$.

The line $y = x$ intersects the circle at two points: approximately $(3.2, 3.2)$ and $(-3.2, -3.2)$.

b. Solve the system of equations.

$$x^2 + y^2 = 4.5^2 \quad (1)$$

$$y = 8 - x \quad (2)$$

Substitute $8 - x$ for y into (1).

$$x^2 + y^2 = 4.5^2$$

$$x^2 + (8 - x)^2 = 4.5^2$$

$$x^2 + 64 - 16x + x^2 = 20.25$$

$$2x^2 - 16x + 43.75 = 0$$

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 + 4ac}}{2a} \\ &= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(43.75)}}{2(2)} \\ &= \frac{16 \pm \sqrt{256 - 350}}{4} \\ &= \frac{16 \pm \sqrt{-94}}{4} \end{aligned}$$

Note:
 $a = 2$
 $b = -16$
 $c = 43.75$

Because $\sqrt{-94}$ is non-real, there are no points of intersection.

c. Solve the system of equations.

$$x^2 + y^2 = 4.5^2 \quad (1)$$

$$x = 4.5 \quad (2)$$

Substitute $x = 4.5$ into (1).

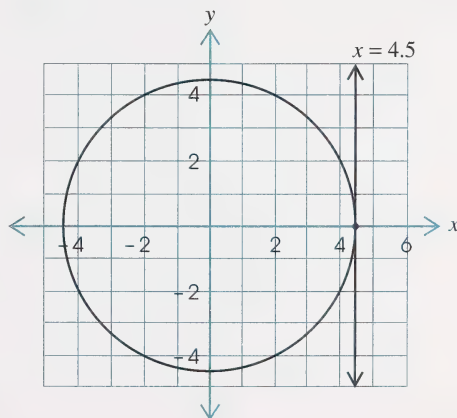
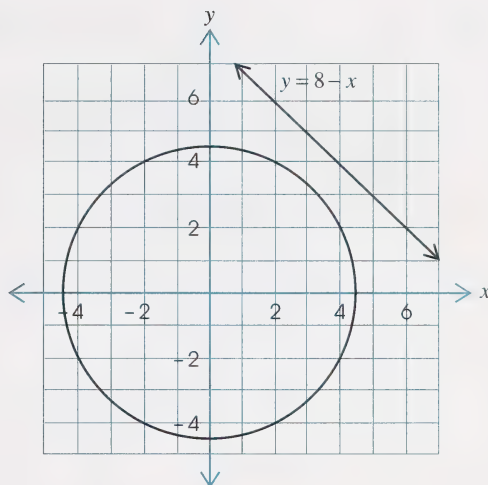
$$x^2 + y^2 = 4.5^2$$

$$4.5^2 + y^2 = 4.5^2$$

$$y^2 = 0$$

$$y = 0$$

The line $x = 4.5$ intersects the circle at one point, $(4.5, 0)$.



Section 1: Activity 4 (continued)

2. a. The maximum number of points in which a line and circle can intersect is 2.
b. The minimum number of points is 0.
3. a. The line $y = 6$ does not intersect the centre circle. The line $y = 6$ is 6 units from the centre of the circle, and the radius is only 4.5 units.
b. The line $y = -4.5$ intersects the circle at one point. The line of $y = -4.5$ is 4.5 units from the centre of the circle, and the radius of the circle is 4.5 units.
c. The line $y = -1$ intersects the circle at two points. The line $y = -1$ is only 1 unit from the centre of the circle, and the radius of the circle is 4.5 units.
d. The line $y = 3x$ intersects the circle at two points. This line passes through the centre of the circle, thus cutting the circumference twice.

4. a. Textbook questions 1, 6, 10, and 17 of “Practice,” p. 488

1. Solve the system of equations.

$$y = x \quad (1)$$

$$x^2 + y^2 = 18 \quad (2)$$

Substitute x for y into (2).

$$x^2 + y^2 = 18$$

$$x^2 + x^2 = 18$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

Substitute $x = 3$ and $x = -3$ into (1).

For $x = 3$,

$$y = x$$

$$= 3$$

For $x = -3$,

$$y = x$$

$$= -3$$

The coordinates of the points of intersection are $(3, 3)$ and $(-3, -3)$.

6. Solve the system of equations.

$$2y = x + 8 \quad (1)$$

$$x^2 + y^2 = 4 \quad (2)$$

Solve (1) for y in terms of x .

$$y = \frac{x+8}{2}$$

Substitute $\frac{x+8}{2}$ for y into (2).

$$x^2 + y^2 = 4$$

$$x^2 + \left(\frac{x+8}{2}\right)^2 = 4$$

$$x^2 + \frac{x^2 + 16x + 64}{4} = 4$$

$$4x^2 + x^2 + 16x + 64 = 16 \quad \leftarrow \text{Multiply both sides by 4.}$$

$$5x^2 + 16x + 48 = 0$$

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-16 \pm \sqrt{16^2 - 4(5)(48)}}{2(5)} \\ &= \frac{-16 \pm \sqrt{256 - 960}}{10} \\ &= \frac{-16 \pm \sqrt{-704}}{10} \end{aligned}$$

Note:

$$\begin{aligned} a &= 5 \\ b &= 16 \\ c &= 48 \end{aligned}$$

Because $\sqrt{-704}$ is non-real, the line $2y = x + 8$ does not intersect the circle $x^2 + y^2 = 4$.

Section 1: Activity 4 (continued)

10. Solve the system of equations.

$$y = x + 2 \quad (1)$$

$$(x+1)^2 + (y-2)^2 = 13 \quad (2)$$

Substitute $x+2$ for y into (2).

$$(x+1)^2 + (y-2)^2 = 13$$

$$(x+1)^2 + (x+2-2)^2 = 13$$

$$(x+1)^2 + x^2 = 13$$

$$x^2 + 2x + 1 + x^2 = 13$$

$$2x^2 + 2x - 12 = 0$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$\therefore x+3 = 0 \quad \text{or} \quad x-2 = 0$$

$$x = -3 \quad \quad \quad x = 2$$

Substitute $x = -3$ and $x = 2$ into (1).

For $x = -3$,

$$y = x + 2$$

$$= -3 + 2$$

$$= -1$$

For $x = 2$,

$$y = x + 2$$

$$= 2 + 2$$

$$= 4$$

The coordinates of the points of intersection are $(-3, -1)$ and $(2, 4)$.

17. Solve the system of equations.

$$2x - y + 3 = 0 \quad (1)$$

$$(x-1)^2 + (y+2)^2 = 36 \quad (2)$$

Solve (1) for y in terms of x .

$$2x - y + 3 = 0$$

$$2x + 3 = y$$

$$y = 2x + 3 \quad (3)$$

Substitute $2x + 3$ for y into (2).

$$(x-1)^2 + (y+2)^2 = 36$$

$$(x-1)^2 + (2x+3+2)^2 = 36$$

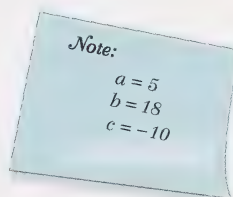
$$(x-1)^2 + (2x+5)^2 = 36$$

$$x^2 - 2x + 1 + 4x^2 + 20x + 25 = 36$$

$$5x^2 + 18x - 10 = 0$$

$$\begin{aligned} \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-18 \pm \sqrt{18^2 - 4(5)(-10)}}{2(5)} \\ &= \frac{-18 \pm \sqrt{324 + 200}}{10} \\ &= \frac{-18 \pm \sqrt{524}}{10} \end{aligned}$$

$$\begin{aligned} \therefore x &= \frac{-18 + \sqrt{524}}{10} \quad \text{or} \quad x = \frac{-18 - \sqrt{524}}{10} \\ &\doteq 0.5 \quad \quad \quad \doteq -4.1 \end{aligned}$$



Substitute $x \doteq 0.5$ and $x \doteq -4.1$ into (3).

For $x \doteq 0.5$,

$$\begin{aligned} y &= 2x + 3 \\ &\doteq 2(0.5) + 3 \\ &\doteq 4.0 \end{aligned}$$

For $x \doteq -4.1$,

$$\begin{aligned} y &= 2x + 3 \\ &\doteq 2(-4.1) + 3 \\ &\doteq -5.2 \end{aligned}$$

The points of intersection are approximately $(0.5, 4.0)$ and $(-4.1, -5.2)$.

Section 1: Activity 4 (continued)

b. Textbook question 67 of “Applications and Problem Solving,” p. 489

67. Compare $(x-15)^2 + (y-40)^2 = 225$ with $(x-h)^2 + (y-k)^2 = r^2$.

$$\therefore (h, k) = (15, 40) \text{ and } r = 15$$

If the line $y = 3x - 5$ passes through the centre $(15, 40)$, the coordinates should satisfy the equation of the line.

LS	RS
y	$3x - 5$
$= 40$	$= 3(15) - 5$
	$= 45 - 5$
	$= 40$
LS	= RS

Therefore, the line $y = 3x - 5$ passes through the centre of the circle $(x-15)^2 + (y-40)^2 = 225$.

5. a. Textbook questions 22, 27, 33, 39, 47, and 61 of “Practice,” p. 489

22. Solve the system of equations.

$$y = 0.5x + 2 \quad (1)$$

$$(x+1)^2 + (y+1)^2 = 9 \quad (2)$$

Substitute $0.5x + 2$ for y into (2).

$$(x+1)^2 + (y+1)^2 = 9$$

$$(x+1)^2 + (0.5x+2+1)^2 = 9$$

$$(x+1)^2 + (0.5x+3)^2 = 9$$

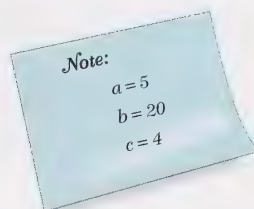
$$x^2 + 2x + 1 + 0.25x^2 + 3x + 9 = 9$$

$$1.25x^2 + 5x + 1 = 0$$

$$125x^2 + 500x + 100 = 0 \quad \leftarrow \text{Multiply both sides by 100.}$$

$$5x^2 + 20x + 4 = 0$$

$$\begin{aligned}
 \therefore x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-20 \pm \sqrt{20^2 - 4(5)(4)}}{2(5)} \\
 &= \frac{-20 \pm \sqrt{400 - 80}}{10} \\
 &= \frac{-20 \pm \sqrt{320}}{10} \\
 &= \frac{-20 \pm 8\sqrt{5}}{10} \\
 &= -2 \pm 0.8\sqrt{5}
 \end{aligned}$$



Substitute $x = -2 + 0.8\sqrt{5}$ and $x = -2 - 0.8\sqrt{5}$ into ①.

For $x = -2 + 0.8\sqrt{5}$,

$$\begin{aligned}
 y &= 0.5x + 2 \\
 &= 0.5(-2 + 0.8\sqrt{5}) + 2 \\
 &= -1 + 0.4\sqrt{5} + 2 \\
 &= 1 + 0.4\sqrt{5}
 \end{aligned}$$

For $x = -2 - 0.8\sqrt{5}$,

$$\begin{aligned}
 y &= 0.5x + 2 \\
 &= 0.5(-2 - 0.8\sqrt{5}) + 2 \\
 &= -1 - 0.4\sqrt{5} + 2 \\
 &= 1 - 0.4\sqrt{5}
 \end{aligned}$$

The line intersects the circle at $(-2 + 0.8\sqrt{5}, 1 + 0.4\sqrt{5})$ and $(-2 - 0.8\sqrt{5}, 1 - 0.4\sqrt{5})$.

Use the distance formula to find the length, d , of the chord. Let

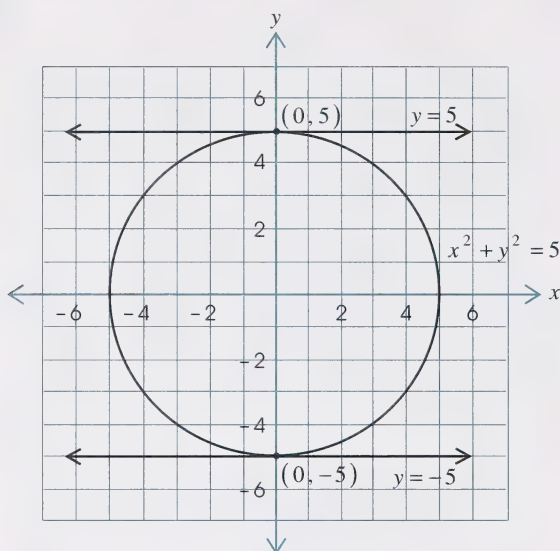
$$(x_1, y_1) = (-2 + 0.8\sqrt{5}, 1 + 0.4\sqrt{5}) \text{ and } (x_2, y_2) = (-2 - 0.8\sqrt{5}, 1 - 0.4\sqrt{5}).$$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[(-2 - 0.8\sqrt{5}) - (-2 + 0.8\sqrt{5})]^2 + [(1 - 0.4\sqrt{5}) - (1 + 0.4\sqrt{5})]^2} \\
 &= \sqrt{(-1.6\sqrt{5})^2 + (-0.8\sqrt{5})^2} \\
 &= \sqrt{12.8 + 3.2} \\
 &= \sqrt{16} \\
 &= 4
 \end{aligned}$$

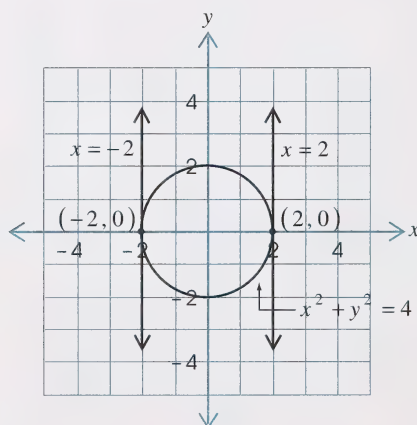
The length of the chord is 4.

Section 1: Activity 4 (continued)

27. The circle $x^2 + y^2 = 25$ intersects the y -axis at $(0, 5)$ and $(0, -5)$. Therefore, the equations of the horizontal tangents are $y = 5$ and $y = -5$.



33. The circle $x^2 + y^2 = 4$ intersects the x -axis at $(2, 0)$ and $(-2, 0)$. Therefore, the equations of the vertical tangents are $x = 2$ and $x = -2$.

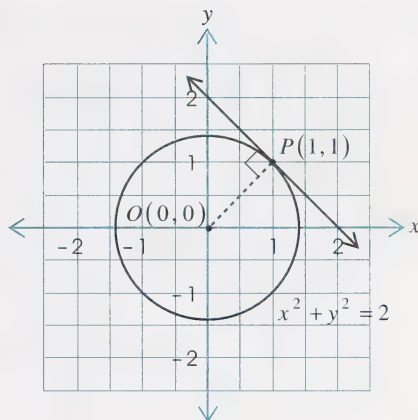


39. The circle $x^2 + y^2 = 2$ has centre $O(0, 0)$ and radius $\sqrt{2}$. The point $P(1, 1)$ lies on the circle.

Find the slope of the radius OP . Let

$$(x_1, y_1) = (0, 0) \text{ and } (x_2, y_2) = (1, 1).$$

$$\begin{aligned} m_{OP} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{1 - 0}{1 - 0} \\ &= 1 \end{aligned}$$



Therefore, the slope of the tangent line is -1 .

Use the point-slope formula to find the equation of the tangent line. Let $m = -1$ and

$$(x_1, y_1) = (1, 1).$$

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 1 &= -1(x - 1) \\ y - 1 &= -x + 1 \\ x + y - 2 &= 0 \end{aligned}$$

The equation of the tangent line is $x + y - 2 = 0$.

47. a. Compare $(x-2)^2 + (y-3)^2 = 20$ with $(x-h)^2 + (y-k)^2 = r^2$. Therefore, the centre of the circle, (h, k) , is $(2, 3)$.
- b. If $(4, 7)$ is on the circle, then its coordinates will satisfy the equation.

LS	RS
$(x-2)^2 + (y-3)^2$	20
$= (4-2)^2 + (7-3)^2$	
$= 2^2 + 4^2$	
$= 4 + 16$	
$= 20$	
LS	RS

Therefore, $(4, 7)$ lies on the circle.

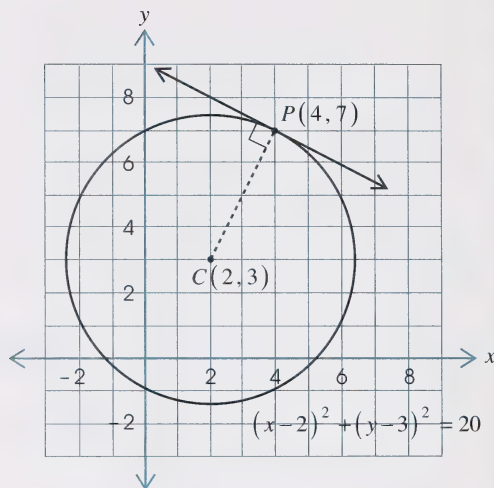
Section 1: Activity 4 (continued)

- c. The circle has centre $C(2, 3)$ and radius $\sqrt{20} = 2\sqrt{5}$. The point $P(4, 7)$ lies on the circle.

Find the slope of the radius CP . Let

$$(x_1, y_1) = (2, 3) \text{ and } (x_2, y_2) = (4, 7).$$

$$\begin{aligned} m_{CP} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{7 - 3}{4 - 2} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$



Therefore, the slope of the tangent line is $-\frac{1}{2}$.

Use the point-slope formula to find the equation of the tangent line. Let $m = -\frac{1}{2}$ and $(x_1, y_1) = (4, 7)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 7 &= -\frac{1}{2}(x - 4) \\ 2y - 14 &= -(x - 4) \quad \leftarrow \text{Multiply both sides by 2.} \\ 2y - 14 &= -x + 4 \\ x + 2y - 18 &= 0 \end{aligned}$$

The equation of the tangent is $x + 2y - 18 = 0$.

61. The circle $(x-2)^2 + (y-2)^2 = 14$ has centre $C(2, 2)$ and radius $\sqrt{14}$.

Draw a tangent from $P(7, 7)$ to the circle.

Let the point of tangency be M .

Draw the segments CM and CP to complete $\triangle CMP$.

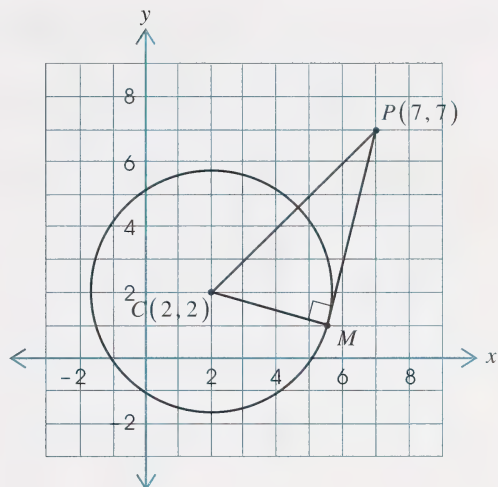
Because CM is a radius, its length is $\sqrt{14}$. Use the distance formula to find CP . Let $(x_1, y_1) = (2, 2)$ and $(x_2, y_2) = (7, 7)$.

$$\begin{aligned} CP &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(7-2)^2 + (7-2)^2} \\ &= \sqrt{5^2 + 5^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \\ &= 5\sqrt{2} \end{aligned}$$

Use the Pythagorean Theorem to find PM .

$$\begin{aligned} (PM)^2 &= (CP)^2 - (CM)^2 \\ (PM)^2 &= (5\sqrt{2})^2 - (\sqrt{14})^2 \\ (PM)^2 &= 50 - 14 \\ (PM)^2 &= 36 \\ PM &= 6 \end{aligned}$$

The length of the tangent is 6.



Section 1: Activity 4 (continued)

b. Textbook questions 68, 69, 70, 78, 87, and 91 of “Applications and Problem Solving,” pp. 489 to 491

68. a. The circle has centre $C(5, 5)$ and radius 5.

Solve the system of equations.

$$x + 3y - 5 = 0 \quad (1)$$

$$(x - 5)^2 + (y - 5)^2 = 25 \quad (2)$$

Solve (1) for x in terms of y .

$$\begin{aligned} x + 3y - 5 &= 0 \\ x &= -3y + 5 \quad (3) \end{aligned}$$

Substitute $-3y + 5$ for x into (2).

$$\begin{aligned} (x - 5)^2 + (y - 5)^2 &= 25 \\ (-3y + 5 - 5)^2 + (y - 5)^2 &= 25 \\ (-3y)^2 + (y - 5)^2 &= 25 \\ 9y^2 + y^2 - 10y + 25 &= 25 \\ 10y^2 - 10y &= 0 \\ y^2 - y &= 0 \\ y(y - 1) &= 0 \end{aligned}$$

$$\therefore y = 0 \quad \text{or} \quad y - 1 = 0$$

$$y = 1$$

Substitute $y = 0$ and $y = 1$ into (3).

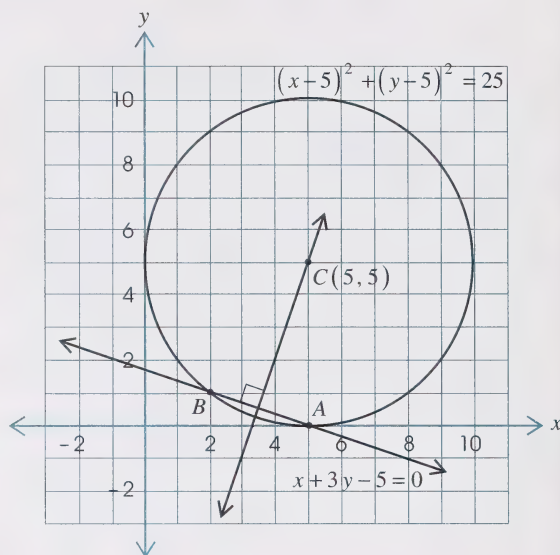
For $y = 0$,

$$\begin{aligned} x &= -3y + 5 \\ &= -3(0) + 5 \\ &= 5 \end{aligned}$$

For $y = 1$,

$$\begin{aligned} x &= -3y + 5 \\ &= -3(1) + 5 \\ &= 2 \end{aligned}$$

The points of intersection are $A(5, 0)$ and $B(2, 1)$, the endpoints of chord AB .



- b. Find the midpoint and slope of AB . Let $(x_1, y_1) = (5, 0)$ and $(x_2, y_2) = (2, 1)$.

$$\begin{aligned} \text{Midpoint of } AB &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) & m_{AB} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \left(\frac{5+2}{2}, \frac{0+1}{2} \right) & &= \frac{1-0}{2-5} \\ &= \left(\frac{7}{2}, \frac{1}{2} \right) & &= -\frac{1}{3} \end{aligned}$$

Because the slope of AB is $-\frac{1}{3}$, the slope of the perpendicular bisector is 3.

Use the point-slope formula to determine the equation of the perpendicular bisector. Let $m = 3$ and $(x_1, y_1) = \left(\frac{7}{2}, \frac{1}{2}\right)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - \frac{1}{2} &= 3\left(x - \frac{7}{2}\right) \\ y - \frac{1}{2} &= 3x - \frac{21}{2} \\ 0 &= 3x - y - \frac{20}{2} \\ 3x - y - 10 &= 0 \end{aligned}$$

If the line $3x - y - 10 = 0$ passes through the centre of the circle, $C(5, 5)$, the coordinates of C will satisfy the equation.

LS	RS
$3x - y - 10$	0
$= 3(5) - 5 - 10$	
$= 15 - 5 - 10$	
$= 0$	
LS	= RS

Therefore, the right bisector of chord AB passes through the centre of the circle.

Section 1: Activity 4 (continued)

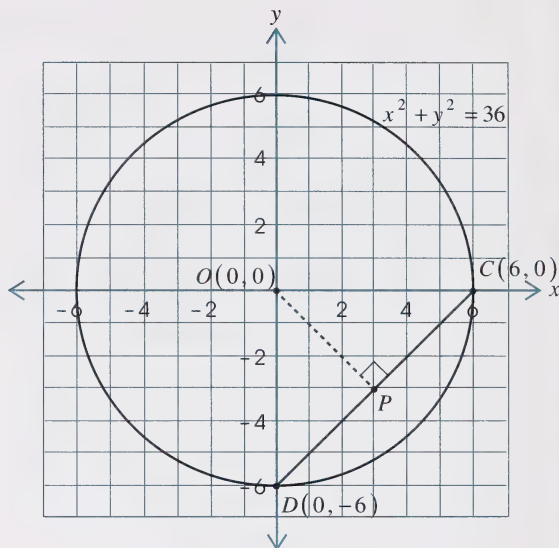
69. The centre of $x^2 + y^2 = 36$ is $(0, 0)$ and the radius is 6.

Determine the slope of CD . Let

$$(x_1, y_1) = (6, 0) \text{ and}$$

$$(x_2, y_2) = (0, -6).$$

$$\begin{aligned} m_{CD} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-6 - 0}{0 - 6} \\ &= \frac{-6}{-6} \\ &= 1 \end{aligned}$$



Because the slope of CD is 1, the slope of the perpendicular bisector is -1 .

Use the point-slope formula to determine the equation of OP . Let $m = -1$ and $(x_1, y_1) = (0, 0)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 0 &= -1(x - 0) \\ y &= -x \end{aligned}$$

To determine if OP bisects chord CD , find the midpoint of CD and verify that this point satisfies the equation of OP .

Let $(x_1, y_1) = (6, 0)$ and $(x_2, y_2) = (0, -6)$.

$$\begin{aligned} \therefore \text{Midpoint of } CD &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{6 + 0}{2}, \frac{0 + (-6)}{2} \right) \\ &= (3, -3) \end{aligned}$$

Check if $(3, -3)$ satisfies $y = -x$.

LS	RS
y	$-x$
$= -3$	$= -3$
LS	= RS

Therefore, the perpendicular from the centre bisects chord CD .

- 70. a.** The centre of circle is $C(-1, 2)$ and the radius is $\sqrt{53}$.

Solve the following system of equations to locate P and Q :

$$5x + 9y = 66 \quad (1)$$

$$(x+1)^2 + (y-2)^2 = 53 \quad (2)$$

Solve (1) for y in terms of x .

$$5x + 9y = 66$$

$$9y = 66 - 5x$$

$$y = \frac{66-5x}{9} \quad (3)$$

Substitute $\frac{66-5x}{9}$ for y into (2).

$$(x+1)^2 + \left(\frac{66-5x}{9} - 2\right)^2 = 53$$

$$(x+1)^2 + \left(\frac{66-5x}{9} - \frac{18}{9}\right)^2 = 53$$

$$(x+1)^2 + \left(\frac{48-5x}{9}\right)^2 = 53$$

$$x^2 + 2x + 1 + \frac{2304 - 480x + 25x^2}{81} = 53$$

$$81x^2 + 162x + 81 + 2304 - 480x + 25x^2 = 4293 \quad \leftarrow \text{Multiply both sides by 81.}$$

$$106x^2 - 318x - 1908 = 0$$

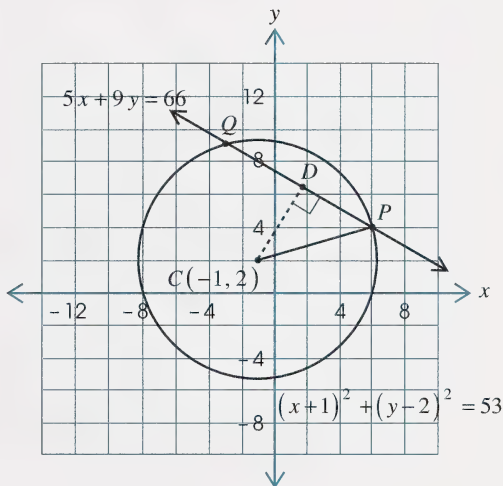
$$x^2 - 3x - 18 = 0$$

$$(x-6)(x+3) = 0$$

$$\therefore x-6 = 0 \quad \text{or} \quad x+3 = 0$$

$$x = 6$$

$$x = -3$$



Section 1: Activity 4 (continued)

Substitute $x = 6$ and $x = -3$ into (3).

For $x = 6$,

$$\begin{aligned} y &= \frac{66 - 5x}{9} \\ &= \frac{66 - 5(6)}{9} \\ &= \frac{36}{9} \\ &= 4 \end{aligned}$$

For $x = -3$,

$$\begin{aligned} y &= \frac{66 - 5x}{9} \\ &= \frac{66 - 5(-3)}{9} \\ &= \frac{81}{9} \\ &= 9 \end{aligned}$$

The line $5x + 9y = 66$ intersects the circle at $P(6, 4)$ and $Q(-3, 9)$.

Use the distance formula to find the length of PQ . Let $(x_1, y_1) = (6, 4)$ and $(x_2, y_2) = (-3, 9)$.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(-3 - 6)^2 + (9 - 4)^2} \\ &= \sqrt{(-9)^2 + 5^2} \\ &= \sqrt{81 + 25} \\ &= \sqrt{106} \end{aligned}$$

The exact length of chord PQ is $\sqrt{106}$.

- b. To find CD , the perpendicular distance from the centre of the circle to chord PQ , use the Pythagorean Theorem.

$$\begin{aligned} DP &= \frac{1}{2}(\text{length of the chord } PQ) & CP &= \text{radius} \\ &= \frac{1}{2}(\sqrt{106}) & &= \sqrt{53} \\ &= \frac{\sqrt{106}}{2} \end{aligned}$$

$$\begin{aligned}
\therefore (CD)^2 &= (CP)^2 - (DP)^2 \\
(CD)^2 &= (\sqrt{53})^2 - \left(\frac{\sqrt{106}}{2}\right)^2 \\
(CD)^2 &= 53 - \frac{106}{4} \\
(CD)^2 &= 53 - 26.5 \\
(CD)^2 &= 26.5 \\
CD &= \sqrt{26.5} \\
&\doteq 5.1
\end{aligned}$$

The distance from the centre of the circle to chord PQ is approximately 5.1.

- 78.** Determine the slope of the radius joining the centre of the circle $O(0, 0)$ and the point $P(8, 15)$. Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = (8, 15)$.

$$\begin{aligned}
m_{OP} &= \frac{y_2 - y_1}{x_2 - x_1} \\
&= \frac{15 - 0}{8 - 0} \\
&= \frac{15}{8}
\end{aligned}$$

Therefore, the slope of the tangent line is $-\frac{8}{15}$.

Use the point-slope formula to find the equation of the tangent at $P(8, 15)$.

Let $m = -\frac{8}{15}$ and $(x_1, y_1) = (8, 15)$.

$$\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 15 &= -\frac{8}{15}(x - 8) \\
y - 15 &= -\frac{8}{15}x + \frac{64}{15} \\
15y - 225 &= -8x + 64 \\
8x + 15y - 289 &= 0
\end{aligned}$$

The equation of the linear path that the car skids along is $8x + 15y - 289 = 0$.

Section 1: Activity 4 (continued)

87. There are four possible circles.

The circle in the first quadrant is

$$(x - \sqrt{10})^2 + (y - \sqrt{10})^2 = 10.$$

The circle in the second quadrant is

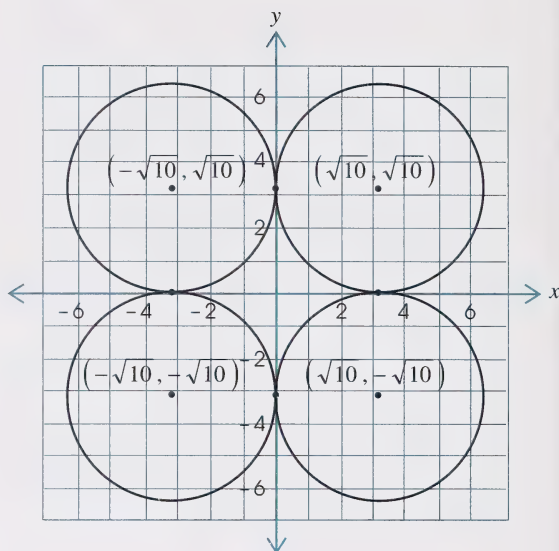
$$(x + \sqrt{10})^2 + (y - \sqrt{10})^2 = 10.$$

The circle in the third quadrant is

$$(x + \sqrt{10})^2 + (y + \sqrt{10})^2 = 10.$$

The circle in the fourth quadrant is

$$(x - \sqrt{10})^2 + (y + \sqrt{10})^2 = 10.$$



91. a. The coordinates of the points of tangency satisfy the equation of the circle.

$$\therefore x^2 + y^2 = 100$$

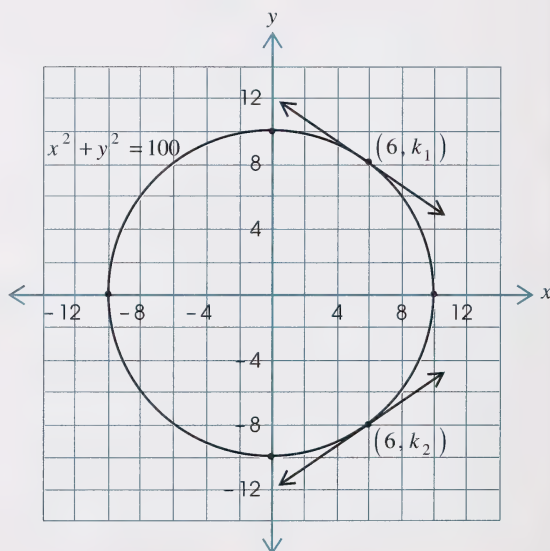
$$6^2 + k^2 = 100$$

$$36 + k^2 = 100$$

$$k^2 = 64$$

$$k = \pm 8$$

The values of k are -8 and 8 .



- b.** First, find the equation of the tangent at $(6, 8)$.

The slope of the radius drawn to $(6, 8)$ is $\frac{8}{6}$ or $\frac{4}{3}$. Therefore, the slope of the tangent is $-\frac{3}{4}$.

Use the point-slope formula to find the equation of the tangent line. Let $m = -\frac{3}{4}$ and $(x_1, y_1) = (6, 8)$.

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -\frac{3}{4}(x - 6)$$

$$y - 8 = -\frac{3}{4}x + \frac{18}{4}$$

$$4y - 32 = -3x + 18$$

$$3x + 4y - 50 = 0$$

Now, find the equation of the tangent at $(6, -8)$.

The slope of the radius drawn to $(6, -8)$ is $-\frac{8}{6}$ or $-\frac{4}{3}$. Therefore, the slope of the tangent line is $\frac{3}{4}$.

Use the point-slope formula to find the equation of the tangent line. Let $m = \frac{3}{4}$ and $(x_1, y_1) = (6, -8)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-8) = \frac{3}{4}(x - 6)$$

$$y + 8 = \frac{3}{4}x - \frac{18}{4}$$

$$4y + 32 = 3x - 18$$

$$0 = 3x - 4y - 50$$

$$3x - 4y - 50 = 0$$

The equations of the tangent lines are $3x + 4y - 50 = 0$ and $3x - 4y - 50 = 0$.

Section 1: Activity 4 (continued)

6. First Move



Second Move



Third Move



Fourth Move



Fifth Move



Section 1: Follow-up Activities

Extra Help

1. Textbook questions 1 to 4, 10, and 21 of “Review,” p. 516

1. Compare the lengths of the sides.

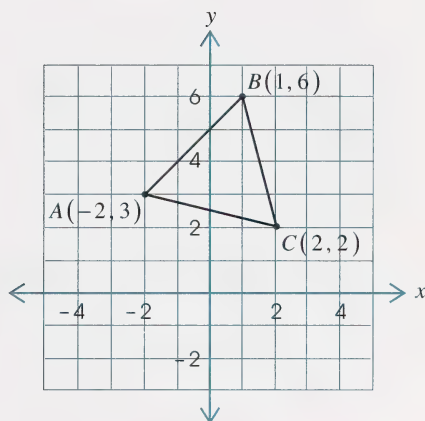
For AB , let $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (1, 6)$.

$$\begin{aligned}
 \therefore AB &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[1 - (-2)]^2 + (6 - 3)^2} \\
 &= \sqrt{3^2 + 3^2} \\
 &= \sqrt{18} \\
 &= 3\sqrt{2}
 \end{aligned}$$

For AC , let $(x_1, y_1) = (-2, 3)$ and $(x_2, y_2) = (2, 2)$.

$$\begin{aligned}
 \therefore AC &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[2 - (-2)]^2 + (2 - 3)^2} \\
 &= \sqrt{4^2 + (-1)^2} \\
 &= \sqrt{17}
 \end{aligned}$$

Because $AB \neq AC$, $\triangle ABC$ is not an equilateral triangle.



Extra Help (continued)

2. Compare the slopes of opposite sides.

For MN , let $(x_1, y_1) = (1, 4)$ and $(x_2, y_2) = (-4, 2)$.

$$\begin{aligned}\therefore m_{MN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 4}{-4 - 1} \\ &= \frac{2}{5}\end{aligned}$$

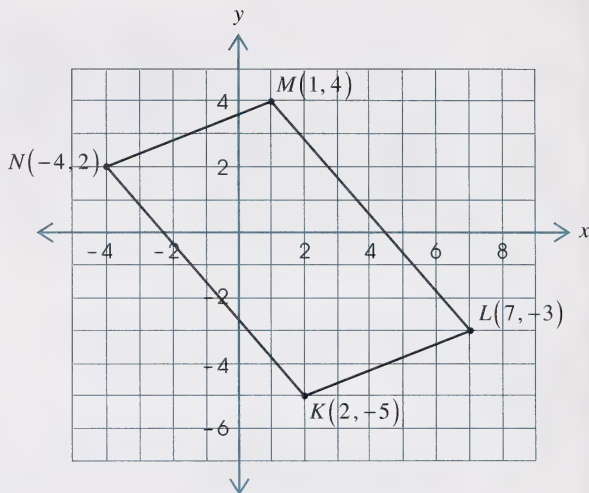
For KL , let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (7, -3)$.

$$\begin{aligned}\therefore m_{KL} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - (-5)}{7 - 2} \\ &= \frac{2}{5}\end{aligned}$$

Because $m_{MN} = m_{KL}$, $MN \parallel KL$.

For KN , let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (-4, 2)$.

$$\begin{aligned}\therefore m_{KN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - (-5)}{-4 - 2} \\ &= -\frac{7}{6}\end{aligned}$$



For LM , let $(x_1, y_1) = (7, -3)$ and $(x_2, y_2) = (1, 4)$.

$$\begin{aligned}\therefore m_{LM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - (-3)}{1 - 7} \\ &= -\frac{7}{6}\end{aligned}$$

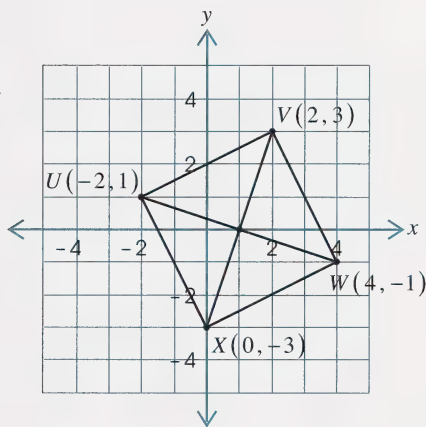
Because $m_{KN} = m_{LM}$, $KN \parallel LM$.

Therefore, $KLMN$ is a parallelogram.

3. Find the slope of each diagonal.

For UW , let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (4, -1)$.

$$\begin{aligned}\therefore m_{UW} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 1}{4 - (-2)} \\ &= \frac{-2}{6} \\ &= -\frac{1}{3}\end{aligned}$$



For VX , let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (0, -3)$.

$$\begin{aligned}\therefore m_{VX} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 3}{0 - 2} \\ &= \frac{-6}{-2} \\ &= 3\end{aligned}$$

Because $m_{UW} \times m_{VX} = -\frac{1}{3}(3) = -1$, $UW \perp VX$.

Extra Help (continued)

Next, find the midpoint of each diagonal.

For UW , let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (4, -1)$.

$$\begin{aligned}\therefore \text{Midpoint of } UW &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 4}{2}, \frac{1 + (-1)}{2} \right) \\ &= \left(\frac{2}{2}, \frac{0}{2} \right) \\ &= (1, 0)\end{aligned}$$

For VX , let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (0, -3)$.

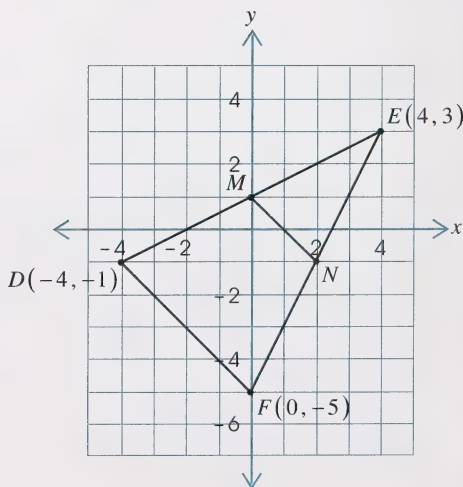
$$\begin{aligned}\therefore \text{Midpoint of } VX &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2 + 0}{2}, \frac{3 + (-3)}{2} \right) \\ &= \left(\frac{2}{2}, \frac{0}{2} \right) \\ &= (1, 0)\end{aligned}$$

Because the diagonals are perpendicular and have a common midpoint, the diagonals perpendicularly bisect each other.

4. a. Determine the lengths of DE and EF .

For DE , let $(x_1, y_1) = (-4, -1)$ and $(x_2, y_2) = (4, 3)$.

$$\begin{aligned}\therefore DE &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[4 - (-4)]^2 + [3 - (-1)]^2} \\ &= \sqrt{8^2 + 4^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} \\ &= 4\sqrt{5}\end{aligned}$$



For EF , let $(x_1, y_1) = (4, 3)$ and $(x_2, y_2) = (0, -5)$.

$$\begin{aligned}
 \therefore EF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(0 - 4)^2 + (-5 - 3)^2} \\
 &= \sqrt{(-4)^2 + (-8)^2} \\
 &= \sqrt{16 + 64} \\
 &= \sqrt{80} \\
 &= 4\sqrt{5}
 \end{aligned}$$

Because $DE = EF$, $\triangle DEF$ is isosceles.

b. Determine the midpoints of DE and EF .

For DE , let $(x_1, y_1) = (-4, -1)$ and $(x_2, y_2) = (4, 3)$.

$$\begin{aligned}
 \therefore \text{Midpoint, } M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{-4 + 4}{2}, \frac{-1 + 3}{2} \right) \\
 &= \left(\frac{0}{2}, \frac{2}{2} \right) \\
 &= (0, 1)
 \end{aligned}$$

For EF , let $(x_1, y_1) = (4, 3)$ and $(x_2, y_2) = (0, -5)$.

$$\begin{aligned}
 \therefore \text{Midpoint, } N &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\
 &= \left(\frac{4 + 0}{2}, \frac{3 + (-5)}{2} \right) \\
 &= \left(\frac{4}{2}, \frac{-2}{2} \right) \\
 &= (2, -1)
 \end{aligned}$$

Extra Help (continued)

Compare the slopes of MN and DF .

For MN , let $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (2, -1)$.

$$\begin{aligned}\therefore m_{MN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-1 - 1}{2 - 0} \\ &= \frac{-2}{2} \\ &= -1\end{aligned}$$

For DF , let $(x_1, y_1) = (-4, -1)$ and $(x_2, y_2) = (0, -5)$.

$$\begin{aligned}\therefore m_{DF} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-5 - (-1)}{0 - (-4)} \\ &= \frac{-4}{4} \\ &= -1\end{aligned}$$

Because $m_{MN} = m_{DF}$, $MN \parallel DF$.

Next, find the lengths of MN and DF .

For MN , let $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (2, -1)$.

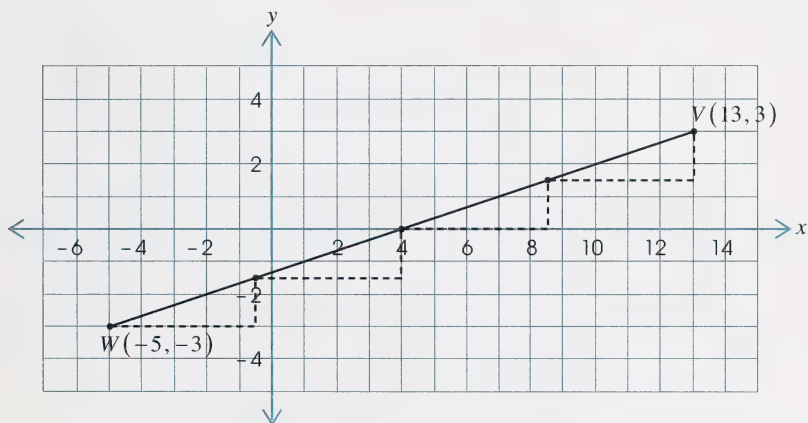
$$\begin{aligned}\therefore MN &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2 - 0)^2 + (-1 - 1)^2} \\ &= \sqrt{2^2 + (-2)^2} \\ &= \sqrt{4 + 4} \\ &= \sqrt{8} \\ &= 2\sqrt{2}\end{aligned}$$

For DF , let $(x_1, y_1) = (-4, -1)$ and $(x_2, y_2) = (0, -5)$.

$$\begin{aligned}
 \therefore DF &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[0 - (-4)]^2 + [-5 - (-1)]^2} \\
 &= \sqrt{4^2 + (-4)^2} \\
 &= \sqrt{16 + 16} \\
 &= \sqrt{32} \\
 &= 4\sqrt{2}
 \end{aligned}$$

Therefore, the line segment joining the midpoints of the equal sides is parallel to the third side and half the length of the third side.

10.



The horizontal distance from V to W is

$$\begin{aligned}
 |x_2 - x_1| &= |-5 - 13| \\
 &= 18
 \end{aligned}$$

The horizontal distance between division points is $\frac{18}{4}$ or 4.5.

The vertical distance from V to W is

$$\begin{aligned}
 |y_2 - y_1| &= |-3 - 3| \\
 &= 6
 \end{aligned}$$

The vertical distance between division points is $\frac{6}{4}$ or 1.5.

Extra Help (continued)

Starting from W , the first division point is 4.5 units to the right and 1.5 units up. Therefore, the coordinates of this division point are $(-5+4.5, -3+1.5) = (-0.5, -1.5)$.

The next division point is $(-5+2(4.5), -3+2(1.5)) = (4, 0)$.

The last division point is $(-5+3(4.5), -3+3(1.5)) = (8.5, 1.5)$.

Therefore, the points that divide VW into four congruent parts are $(-0.5, -1.5)$, $(4, 0)$, and $(8.5, 1.5)$.

21. The shortest distance is the perpendicular distance PQ from $P(-1, -3)$ to the line $x + 3y - 9 = 0$.

Determine the slope of the line.

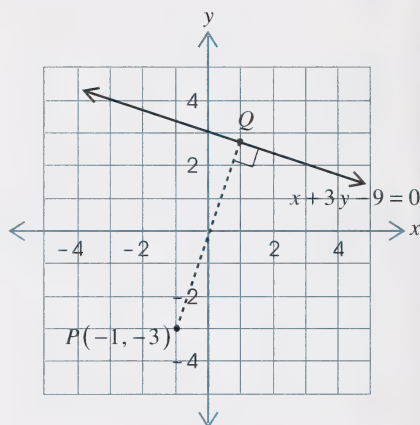
$$x + 3y - 9 = 0$$

$$3y = -x + 9$$

$$y = -\frac{1}{3}x + 3$$

$$y = mx + b$$

$$\therefore m = -\frac{1}{3}$$



Because the slope of the line is $-\frac{1}{3}$, the slope of the perpendicular PQ is 3.

To find the equation of PQ , use the point-slope formula. Let $m = 3$ and $(x_1, y_1) = (-1, -3)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 3[x - (-1)]$$

$$y + 3 = 3(x + 1)$$

$$y + 3 = 3x + 3$$

$$y = 3x$$

To find the coordinates of Q , solve the following system of equations:

$$y = 3x \quad (1)$$

$$x + 3y - 9 = 0 \quad (2)$$

Substitute $3x$ for y into (2).

$$x + 3y - 9 = 0$$

$$x + 3(3x) - 9 = 0$$

$$10x = 9$$

$$x = \frac{9}{10}$$

$$= 0.9$$

Substitute $x = 0.9$ into (1).

$$y = 3x$$

$$= 3(0.9)$$

$$= 2.7$$

The coordinates of Q are $(0.9, 2.7)$.

Use the distance formula to find PQ . Let $(x_1, y_1) = (-1, -3)$ and $(x_2, y_2) = (0.9, 2.7)$.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{[0.9 - (-1)]^2 + [2.7 - (-3)]^2} \\ &= \sqrt{1.9^2 + 5.7^2} \\ &\doteq 6.0 \end{aligned}$$

The shortest distance from $(-1, -3)$ to $x + 3y - 9 = 0$ is approximately 6.0.

Extra Help (continued)

2. Textbook questions 2, 10, and 16 of “Chapter Check,” p. 518

2. Find the midpoint of each side.

For K , the midpoint of RS , let $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (-2, 5)$.

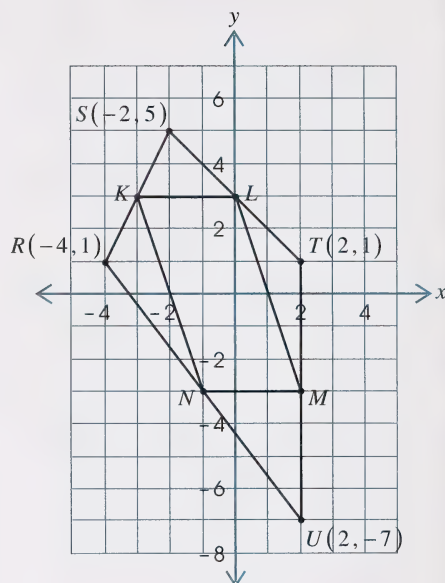
$$\begin{aligned}\therefore K &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-4 + (-2)}{2}, \frac{1 + 5}{2} \right) \\ &= \left(\frac{-6}{2}, \frac{6}{2} \right) \\ &= (-3, 3)\end{aligned}$$

For L , the midpoint of ST , let $(x_1, y_1) = (-2, 5)$ and $(x_2, y_2) = (2, 1)$.

$$\begin{aligned}\therefore L &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-2 + 2}{2}, \frac{5 + 1}{2} \right) \\ &= \left(\frac{0}{2}, \frac{6}{2} \right) \\ &= (0, 3)\end{aligned}$$

For M , the midpoint of TU , let $(x_1, y_1) = (2, 1)$ and $(x_2, y_2) = (2, -7)$.

$$\begin{aligned}\therefore M &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{2 + 2}{2}, \frac{1 + (-7)}{2} \right) \\ &= \left(\frac{4}{2}, \frac{-6}{2} \right) \\ &= (2, -3)\end{aligned}$$



For N , the midpoint of RU , let $(x_1, y_1) = (-4, 1)$ and $(x_2, y_2) = (2, -7)$.

$$\begin{aligned}\therefore N &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-4 + 2}{2}, \frac{1 + (-7)}{2} \right) \\ &= \left(\frac{-2}{2}, \frac{-6}{2} \right) \\ &= (-1, -3)\end{aligned}$$

Next, find the slopes of the opposite sides of $KLMN$.

For KL , let $(x_1, y_1) = (-3, 3)$ and $(x_2, y_2) = (0, 3)$.

$$\begin{aligned}\therefore m_{KL} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{3 - 3}{0 - (-3)} \\ &= 0\end{aligned}$$

For MN , let $(x_1, y_1) = (2, -3)$ and $(x_2, y_2) = (-1, -3)$.

$$\begin{aligned}\therefore m_{MN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - (-3)}{-1 - 2} \\ &= 0\end{aligned}$$

Because $m_{KL} = m_{MN}$, $KL \parallel MN$.

For LM , let $(x_1, y_1) = (0, 3)$ and $(x_2, y_2) = (2, -3)$.

$$\begin{aligned}\therefore m_{LM} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 3}{2 - 0} \\ &= \frac{-6}{2} \\ &= -3\end{aligned}$$

Extra Help (continued)

For KN , let $(x_1, y_1) = (-3, 3)$ and $(x_2, y_2) = (-1, -3)$.

$$\begin{aligned}\therefore m_{KN} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 3}{-1 - (-3)} \\ &= \frac{-6}{2} \\ &= -3\end{aligned}$$

Because $m_{LM} = m_{KN}$, $LM \parallel KN$.

Therefore, $KLMN$ is a parallelogram.

10. The shortest distance is the perpendicular distance OA from $O(0, 0)$ to the line $y = x + 3$. To find the length of OA , find the coordinates of A .

The slope of $y = x + 3$ is 1. Therefore, because OA and $y = x + 3$ are perpendicular, the slope of OA is -1 .

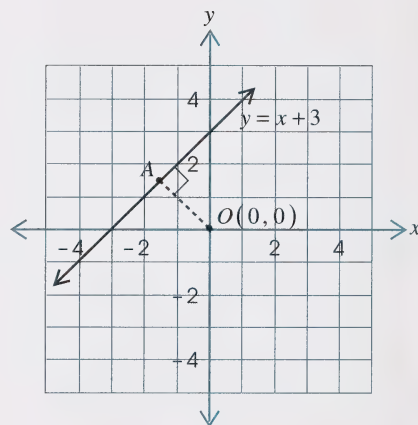
Use the point-slope formula to find the equation for OA .

Let $m = -1$ and $(x_1, y_1) = (0, 0)$.

$$\begin{aligned}y - y_1 &= m(x - x_1) \\ y - 0 &= -1(x - 0) \\ y &= -x\end{aligned}$$

To find the coordinates of A , solve the following system of equations:

$$\begin{aligned}y &= -x & (1) \\ y &= x + 3 & (2)\end{aligned}$$



Substitute $x+3$ for y into ①.

$$x+3=-x$$

$$2x=-3$$

$$x=-\frac{3}{2}$$

Substitute $x=-\frac{3}{2}$ into ①.

$$y=-x$$

$$=-\left(-\frac{3}{2}\right)$$

$$=\frac{3}{2}$$

The coordinates of A are $\left(-\frac{3}{2}, \frac{3}{2}\right)$.

Find the distance from O to A . Let $(x_1, y_1) = (0, 0)$ and $(x_2, y_2) = \left(-\frac{3}{2}, \frac{3}{2}\right)$.

$$\begin{aligned} OA &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(-\frac{3}{2} - 0\right)^2 + \left(\frac{3}{2} - 0\right)^2} \\ &= \sqrt{\frac{9}{4} + \frac{9}{4}} \\ &= \sqrt{\frac{18}{4}} \\ &= \frac{3\sqrt{2}}{2} \end{aligned}$$

The shortest distance from the origin to the line $y = x + 3$ is $\frac{3\sqrt{2}}{2}$.

Extra Help (continued)

16. The vertical distance between the lines is the difference between their y -intercepts. The y -intercept of $y = 4x + 1$ is 1, and the y -intercept of $y = 4x - 2$ is -2 .

$$\begin{aligned}\therefore |y_2 - y_1| &= |-2 - 1| \\ &= 3\end{aligned}$$

The vertical distance between the lines is 3.

The horizontal distance between the lines is the difference between their x -intercepts.

To find the x -intercepts, let $y = 0$.

$$\text{For } y = 4x + 1,$$

$$0 = 4x + 1$$

$$-1 = 4x$$

$$x = -\frac{1}{4}$$

$$\text{For } y = 4x - 2,$$

$$0 = 4x - 2$$

$$2 = 4x$$

$$x = \frac{2}{4}$$

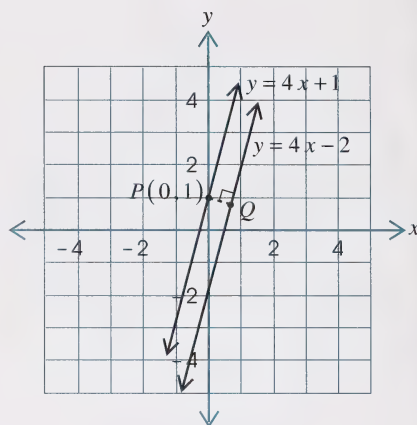
$$= \frac{1}{2}$$

$$\begin{aligned}\therefore |x_2 - x_1| &= \left| \frac{1}{2} - \left(-\frac{1}{4}\right) \right| \\ &= \frac{3}{4}\end{aligned}$$

The horizontal distance between the lines is $\frac{3}{4}$.

To find the shortest distance between the lines, pick a point on $y = 4x + 1$, say $P(0, 1)$, and determine the perpendicular distance PQ to line $y = 4x - 2$.

To locate Q , determine the equation of the line through PQ . The slope of $y = 4x - 2$ is 4. Therefore, the slope of PQ is $-\frac{1}{4}$.



Use the point-slope formula. Let $m = -\frac{1}{4}$ and $(x_1, y_1) = (0, 1)$.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{4}(x - 0)$$

$$y - 1 = -\frac{1}{4}x$$

$$y = -\frac{1}{4}x + 1$$

Solve the system of equations.

$$y = -\frac{1}{4}x + 1 \quad (1)$$

$$y = 4x - 2 \quad (2)$$

Substitute $4x - 2$ for y into (1).

$$4x - 2 = -\frac{1}{4}x + 1$$

$$16x - 8 = -x + 4$$

$$17x = 12$$

$$x = \frac{12}{17}$$

Substitute $x = \frac{12}{17}$ into (2).

$$y = 4x - 2$$

$$= 4\left(\frac{12}{17}\right) - 2$$

$$= \frac{48}{17} - \frac{34}{17}$$

$$= \frac{14}{17}$$

The coordinates of Q are $\left(\frac{12}{17}, \frac{14}{17}\right)$.

Extra Help (continued)

Use the distance formula to find PQ . Let $(x_1, y_1) = (0, 1)$ and $(x_2, y_2) = (\frac{12}{17}, \frac{14}{17})$.

$$\begin{aligned} PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\left(\frac{12}{17} - 0\right)^2 + \left(\frac{14}{17} - 1\right)^2} \\ &= \sqrt{\left(\frac{12}{17}\right)^2 + \left(-\frac{3}{17}\right)^2} \\ &\doteq 0.73 \end{aligned}$$

The shortest distance between the two lines is approximately 0.73.

Enrichment

- Write the given line in standard form, $Ax + By + C = 0$.

$$2x - 3y = 1$$

$$2x - 3y - 1 = 0$$

$$\therefore A = 2, B = -3, C = -1, \text{ and } (x_1, y_1) = (0, 4)$$

$$\begin{aligned} \therefore d &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|2(0) + (-3)(4) + (-1)|}{\sqrt{2^2 + (-3)^2}} \\ &= \frac{|0 - 12 - 1|}{\sqrt{4 + 9}} \\ &= \frac{13}{\sqrt{13}} \\ &= \sqrt{13} \end{aligned}$$

The shortest distance from $(0, 4)$ to $2x - 3y = 1$ is $\sqrt{13}$.

2. Write the given line in standard form, $Ax + By + C = 0$.

$$2x + 3y = 10$$

$$2x + 3y - 10 = 0$$

$$\therefore A = 2, B = 3, C = -10, \text{ and } (x_1, y_1) = (-3, -4)$$

$$\begin{aligned} \therefore d &= \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}} \\ &= \frac{|2(-3) + 3(-4) + (-10)|}{\sqrt{2^2 + 3^2}} \\ &= \frac{|-6 - 12 - 10|}{\sqrt{4 + 9}} \\ &= \frac{28}{\sqrt{13}} \\ &= \frac{28\sqrt{13}}{13} \end{aligned}$$

The shortest distance from $(-3, -4)$ to $2x + 3y = 10$ is $\frac{28\sqrt{13}}{13}$.

Section 2: Activity 1

1. a. Textbook questions a. and b. of “Explore: Use a Diagram,” p. 493

- a. Segment CD represents the height of the lighthouse.
- b. Segment BC represents the height of the foot of the lighthouse above sea level.

- b. Textbook questions 1 to 5 of “Inquire,” p. 493

$$1. \quad \text{a.} \quad \tan \angle BAC = \frac{BC}{AB}$$

$$BC = AB \tan \angle BAC$$

$$\text{b.} \quad BC = 1000 \tan 4.3^\circ$$

$$\doteq 75.2 \text{ m}$$

$$2. \quad \text{a.} \quad \tan \angle BAD = \frac{BD}{AB}$$

$$BD = AB \tan \angle BAD$$

$$\text{b.} \quad BD = 1000 \tan 4.9^\circ$$

$$\doteq 85.7 \text{ m}$$

Section 2: Activity 1 (continued)

$$\begin{aligned}
 3. \quad CD &= BD - BC \\
 &\doteq 85.7 - 75.2 \\
 &\doteq 10.5
 \end{aligned}$$

The height of the lighthouse is about 10.5 m.

4. Segment AC can be found from $\triangle ABC$ using the cosine ratio.
Segment CD can be found from $\triangle ACD$ using the Sine Law.

5. The angle of depression of point A from the top of the lighthouse is 4.9° . The angle of depression is equal to the angle of elevation to the top of the lighthouse from point A .

2. a. Textbook questions 2, 3, 9, 23, 25, 33, and 39 of “Practice,” pp. 497 and 498

$$\begin{aligned}
 2. \quad \sin 27^\circ &= \frac{q}{6.2} \\
 q &= 6.2 \sin 27^\circ \\
 &\doteq 2.8
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \cos 65^\circ &= \frac{m}{19} \\
 m &= 19 \cos 65^\circ \\
 &\doteq 8.0
 \end{aligned}$$

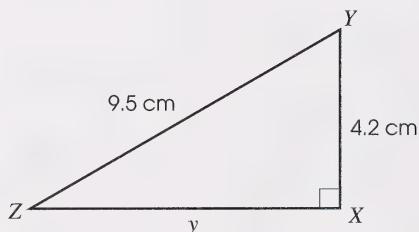
$$\begin{aligned}
 9. \quad \tan E &= \frac{5.8}{7.4} \\
 \angle E &\doteq 38.1^\circ
 \end{aligned}$$

23. Use the Pythagorean Theorem to find y .

$$\begin{aligned}
 y^2 &= 9.5^2 - 4.2^2 \\
 y &= \sqrt{9.5^2 - 4.2^2} \\
 &\doteq 8.5 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 \sin Z &= \frac{4.2}{9.5} \\
 \angle Z &\doteq 26.2^\circ
 \end{aligned}$$

$$\begin{aligned}
 \angle Y &= 90^\circ - \angle Z \\
 &\doteq 90^\circ - 26.2^\circ \\
 &\doteq 63.8^\circ
 \end{aligned}$$

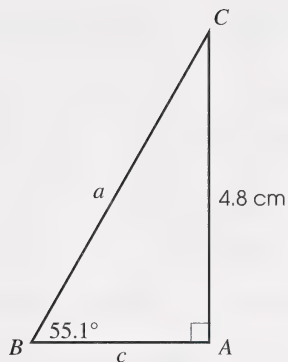


In $\triangle XYZ$, $y \doteq 8.5$ cm, $\angle Y \doteq 63.8^\circ$, and $\angle Z \doteq 26.2^\circ$.

$$\begin{aligned}
 25. \quad \angle C &= 90^\circ - \angle B \\
 &= 90^\circ - 55.1^\circ \\
 &= 34.9^\circ
 \end{aligned}$$

$$\begin{aligned}
 \sin 55.1^\circ &= \frac{4.8}{a} \\
 a \sin 55.1^\circ &= 4.8 \\
 a &= \frac{4.8}{\sin 55.1^\circ} \\
 &\doteq 5.9
 \end{aligned}$$

$$\begin{aligned}
 \tan 55.1^\circ &= \frac{4.8}{c} \\
 c \tan 55.1^\circ &= 4.8 \\
 c &= \frac{4.8}{\tan 55.1^\circ} \\
 &\doteq 3.3
 \end{aligned}$$



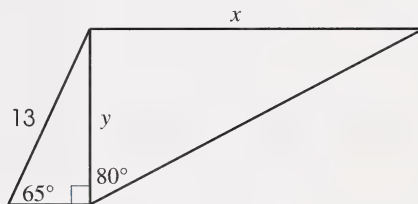
In $\triangle ABC$, $a \doteq 5.9$ m, $c \doteq 3.3$ m, and $\angle C = 34.9^\circ$.

33. First, find y .

$$\begin{aligned}
 \sin 65^\circ &= \frac{y}{13} \\
 y &= 13 \sin 65^\circ
 \end{aligned}$$

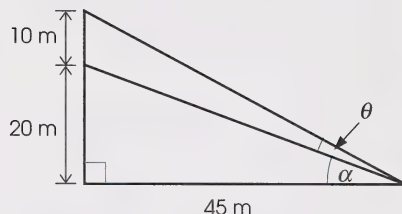
Now, find x .

$$\begin{aligned}
 \tan 80^\circ &= \frac{x}{y} \\
 x &= y \tan 80^\circ \\
 &= 13(\sin 65^\circ)(\tan 80^\circ) \\
 &\doteq 66.8
 \end{aligned}$$



Section 2: Activity 1 (continued)

$$\begin{aligned}
 39. \quad \tan \alpha &= \frac{20}{45} & \tan(\alpha + \theta) &= \frac{10+20}{45} \\
 \angle \alpha &\doteq 24.0^\circ & \angle(\alpha + \theta) &\doteq 33.7^\circ \\
 \therefore \angle \theta &\doteq 33.7^\circ - 24.0^\circ \\
 &\doteq 9.7^\circ
 \end{aligned}$$

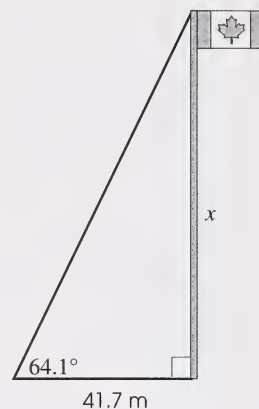


b. Textbook questions 43 and 44 of “Applications and Problem Solving,” p. 498

43. Let
- x
- be the height of the flagpole.

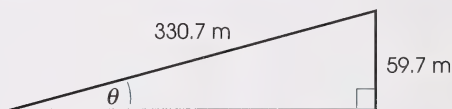
$$\begin{aligned}
 \therefore \tan 64.1^\circ &= \frac{x}{41.7} \\
 x &= 41.7 \tan 64.1^\circ \\
 &\doteq 85.9
 \end{aligned}$$

The height of the flagpole is approximately 85.9 m.



44. Let
- θ
- be the angle of elevation.

$$\begin{aligned}
 \therefore \sin \theta &= \frac{59.7}{330.7} \\
 \angle \theta &\doteq 10.4^\circ
 \end{aligned}$$

The angle of elevation of the top of the escalator is approximately 10.4° .

3. a. Textbook questions 11, 16, 20, 27, 31, and 41 of “Practice,” pp. 497 and 498

11. Use the Sine Law to find
- x
- .

$$\begin{aligned}
 \frac{x}{\sin X} &= \frac{z}{\sin Z} \\
 \frac{x}{\sin 47^\circ} &= \frac{13}{\sin 58^\circ} \\
 x &= \frac{13 \sin 47^\circ}{\sin 58^\circ} \\
 &\doteq 11.2
 \end{aligned}$$

16. Use the Cosine Law to find r .

$$r^2 = p^2 + q^2 - 2pq \cos 101^\circ$$

$$r^2 = 12.8^2 + 8.3^2 - 2(12.8)(8.3) \cos 101^\circ$$

$$r^2 \doteq 273.27$$

$$r \doteq 16.5$$

20. Use the Sine Law to find $\angle L$.

$$\frac{\sin L}{\ell} = \frac{\sin N}{n}$$

$$\frac{\sin L}{16.8} = \frac{\sin 136^\circ}{27.3}$$

$$\sin L = \frac{16.8 \sin 136^\circ}{27.3}$$

$$\angle L \doteq 25.3^\circ$$

27. $\angle B = 180^\circ - (\angle A + \angle C)$

$$= 180^\circ - (84^\circ + 40^\circ)$$

$$= 180^\circ - 124^\circ$$

$$= 56^\circ$$

Use the Sine Law to find b and c .

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 56^\circ} = \frac{5.6}{\sin 84^\circ}$$

$$b = \frac{5.6 \sin 56^\circ}{\sin 84^\circ}$$

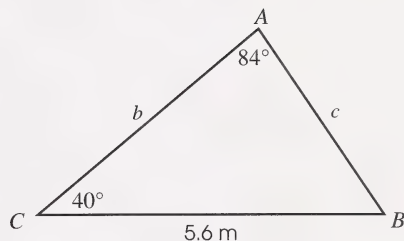
$$\doteq 4.7$$

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{c}{\sin 40^\circ} = \frac{5.6}{\sin 84^\circ}$$

$$c = \frac{5.6 \sin 40^\circ}{\sin 84^\circ}$$

$$\doteq 3.6$$

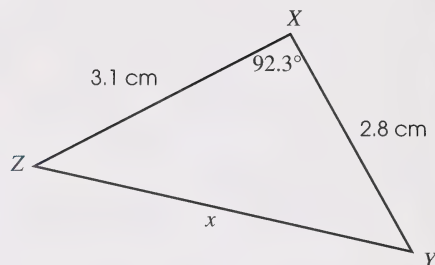


In $\triangle ABC$, $b \doteq 4.7$ m, $c \doteq 3.6$ m, and $\angle B = 56^\circ$.

Section 2: Activity 1 (continued)

31. Use the Cosine Law to find
- x
- .

$$\begin{aligned}
 x^2 &= y^2 + z^2 - 2yz \cos 92.3^\circ \\
 x^2 &= 3.1^2 + 2.8^2 - 2(3.1)(2.8) \cos 92.3^\circ \\
 x &\doteq 4.259\,892\,947 \\
 &\doteq 4.3
 \end{aligned}$$

Use the Sine Law to find $\angle Y$.

$$\begin{aligned}
 \frac{\sin Y}{y} &= \frac{\sin X}{x} \\
 \frac{\sin Y}{3.1} &= \frac{\sin 92.3^\circ}{4.259\,892\,947} \quad \leftarrow \text{Use the answer for } x \text{ before rounding.} \\
 \sin Y &= \frac{3.1 \sin 92.3^\circ}{4.259\,892\,947} \\
 \angle Y &\doteq 46.6^\circ
 \end{aligned}$$

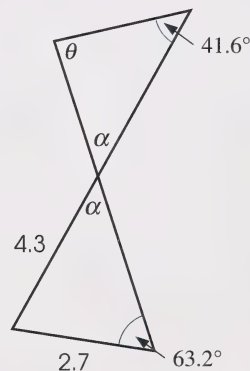
$$\begin{aligned}
 \therefore \angle Z &= 180^\circ - (\angle X + \angle Y) \\
 &\doteq 180^\circ - (92.3^\circ + 46.6^\circ) \\
 &\doteq 180^\circ - 138.9^\circ \\
 &\doteq 41.1^\circ
 \end{aligned}$$

 $\triangle XYZ$, $x \doteq 4.3$ cm, $\angle Y \doteq 46.6^\circ$, and $\angle Z \doteq 41.1^\circ$.

41. Find
- $\angle \alpha$
- using the Sine Law.

$$\begin{aligned}
 \frac{\sin \alpha}{2.7} &= \frac{\sin 63.2^\circ}{4.3} \\
 \sin \alpha &= \frac{2.7 \sin 63.2^\circ}{4.3} \\
 \angle \alpha &= 34.1^\circ
 \end{aligned}$$

$$\begin{aligned}
 \therefore \angle \theta &= 180^\circ - (\angle \alpha + 41.6^\circ) \\
 &\doteq 180^\circ - (34.1^\circ + 41.6^\circ) \\
 &\doteq 180^\circ - 75.7^\circ \\
 &\doteq 104.3^\circ
 \end{aligned}$$



b. Textbook questions 48 and 53 of “Applications and Problem Solving,” p. 499

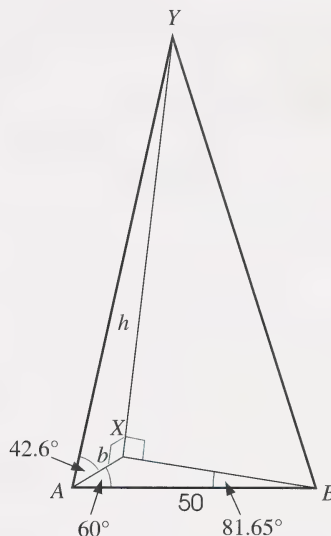
$$\begin{aligned}
 48. \quad \angle AXB &= 180^\circ - (60^\circ + 81.65^\circ) \\
 &= 180^\circ - 141.65^\circ \\
 &= 38.35^\circ
 \end{aligned}$$

Use the Sine Law and $\triangle ABX$ to find b .

$$\begin{aligned}
 \frac{b}{\sin 81.65^\circ} &= \frac{50}{\sin 38.35^\circ} \\
 b &= \frac{50 \sin 81.65^\circ}{\sin 38.35^\circ}
 \end{aligned}$$

Use $\triangle AXY$ to find h .

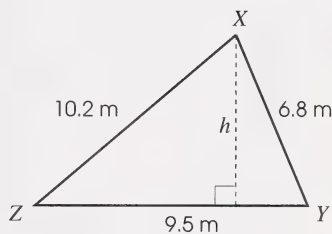
$$\begin{aligned}
 \tan 42.6^\circ &= \frac{h}{b} \\
 h &= b \tan 42.6^\circ \\
 &= \frac{50(\sin 81.65^\circ)(\tan 42.6^\circ)}{\sin 38.35^\circ} \\
 &\doteq 73
 \end{aligned}$$



The height of the Peace Tower is approximately 73 m.

53. Use the Cosine Law to find $\angle Z$.

$$\begin{aligned}
 z^2 &= x^2 + y^2 - 2xy \cos Z \\
 2xy \cos Z &= x^2 + y^2 - z^2 \\
 \cos Z &= \frac{x^2 + y^2 - z^2}{2xy} \\
 \cos Z &= \frac{9.5^2 + 10.2^2 - 6.8^2}{2(9.5)(10.2)} \\
 \angle Z &\doteq 40.19^\circ
 \end{aligned}$$



Section 2: Activity 1 (continued)

Draw an altitude, h , from X .

$$\therefore \frac{h}{10.2} = \sin Z$$

$$h \doteq 10.2 \sin 40.19^\circ$$

$$\doteq 6.58$$

$$\therefore \text{Area of } \triangle XYZ = \frac{1}{2} \text{base} \times \text{height}$$

$$\doteq \frac{1}{2}(9.5)(6.58)$$

$$\doteq 31.3$$

The area of $\triangle XYZ$ is approximately 31.3 m^2 .

4. First, find $\angle \alpha$.

$$\begin{aligned}\angle \alpha &= 25^\circ - 20^\circ \\ &= 5^\circ\end{aligned}$$

Next, find h .

$$\frac{h}{\sin 20^\circ} = \frac{1}{\sin 5^\circ}$$

$$h = \frac{\sin 20^\circ}{\sin 5^\circ}$$

Now, find d .

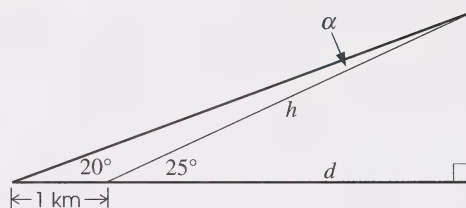
$$\cos 25^\circ = \frac{d}{h}$$

$$d = h \cos 25^\circ$$

$$= \frac{\sin 20^\circ}{\sin 5^\circ} (\cos 25^\circ)$$

$$= \frac{(\sin 20^\circ)(\cos 25^\circ)}{\sin 5^\circ}$$

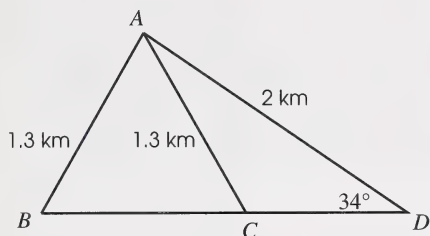
$$\doteq 3.6$$



The distance between you and the mountain is approximately 3.6 km.

Section 2: Activity 2

1. a. Textbook question “Explore: Draw a Diagram,” p. 502



- b. Textbook questions 1 to 5 of “Inquire,” p. 502

$$1. \quad \frac{\sin \angle ABD}{2} = \frac{\sin 34^\circ}{1.3}$$

$$\sin \angle ABD = \frac{2 \sin 34^\circ}{1.3}$$

$$2. \quad \frac{\sin \angle ACD}{2} = \frac{\sin 34^\circ}{1.3}$$

$$\sin \angle ACD = \frac{2 \sin 34^\circ}{1.3}$$

3. a. The values of $\sin \angle ABD$ and $\sin \angle ACD$ are equal.
b. The calculator results would be equal.
4. a. $\angle ABD$ is acute.
b. $\angle ACD$ is obtuse.
c. $\angle ABD$ cannot equal $\angle ACD$.
d. The calculated angle is about 59.3° . This is the measure of $\angle ABD$.
5. The word “ambiguous” is appropriate because there are two distinct angles that have the same sine. You cannot tell the difference between the angles using the sine value only.

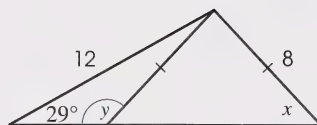
Section 2: Activity 2 (continued)

2. a. Textbook questions 1, 2, 7, 8, 15, 17, and 23 of “Practice,” p. 510

$$1. \quad \frac{\sin x}{12} = \frac{\sin 29^\circ}{8}$$

$$\sin x = \frac{12 \sin 29^\circ}{8}$$

$$\angle x \doteq 46.7^\circ$$



$$\angle y = 180^\circ - \angle x$$

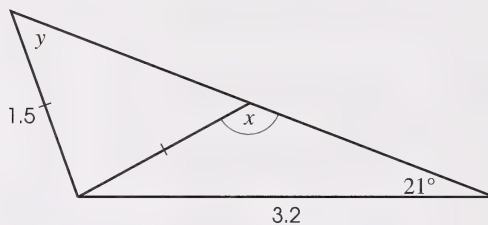
$$\doteq 180^\circ - 46.7^\circ$$

$$\doteq 133.3^\circ$$

$$2. \quad \frac{\sin y}{3.2} = \frac{\sin 21^\circ}{1.5}$$

$$\sin y = \frac{3.2 \sin 21^\circ}{1.5}$$

$$\angle y \doteq 49.9^\circ$$



$$\angle x = 180^\circ - \angle y$$

$$\doteq 180^\circ - 49.9^\circ$$

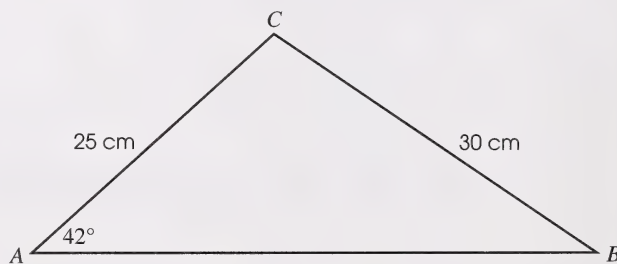
$$\doteq 130.1^\circ$$

7. Draw the diagram. Because $a > b$, there is one triangle and, therefore, one solution.Use the Sine Law to find $\angle B$.

$$\frac{\sin B}{25} = \frac{\sin 42^\circ}{30}$$

$$\sin B = \frac{25 \sin 42^\circ}{30}$$

$$\angle B \doteq 33.9^\circ$$



$$\therefore \angle C = 180^\circ - (\angle A + \angle B)$$

$$\doteq 180^\circ - (42^\circ + 33.9^\circ)$$

$$\doteq 180^\circ - 75.9^\circ$$

$$\doteq 104.1^\circ$$

8. Draw a diagram.

$$c \sin B = 30 \sin 27^\circ \\ \doteq 13.6$$

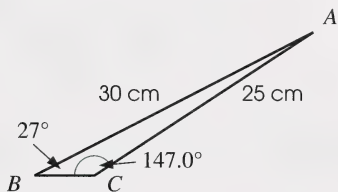
Because $c \sin B < b < c$, there are two locations for point C , two triangles ABC , and two solutions.

First, find the two possible measures for $\angle ACB$.

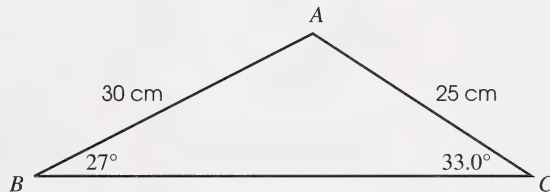
$$\frac{\sin C}{30} = \frac{\sin 27^\circ}{25} \\ \sin C = \frac{30 \sin 27^\circ}{25}$$

$$\therefore \angle C \doteq 33.0^\circ \quad \text{or} \quad \angle C \doteq 147.0^\circ$$

Find the angles for both triangles ABC .



$$\angle A \doteq 180^\circ - (27^\circ + 147.0^\circ) \\ \doteq 6.0^\circ$$



$$\angle A \doteq 180^\circ - (27^\circ + 33.0^\circ) \\ \doteq 120.0^\circ$$

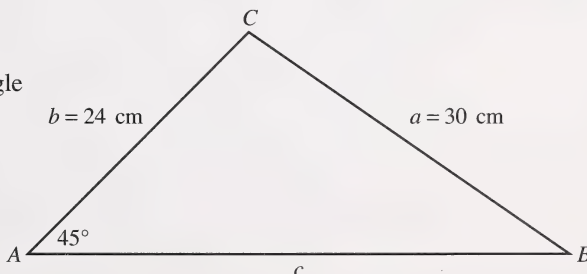
The solutions are $\angle A \doteq 6.0^\circ$ and $\angle C \doteq 147.0^\circ$ or $\angle A \doteq 120.0^\circ$ and $\angle C \doteq 33.0^\circ$.

15. Draw a diagram.

Because $a > b$, there is only one triangle and one solution.

Use the Sine Law to find $\angle B$.

$$\frac{\sin B}{24} = \frac{\sin 45^\circ}{30} \\ \sin B = \frac{24 \sin 45^\circ}{30} \\ \angle B \doteq 34.4^\circ$$



Section 2: Activity 2 (continued)

$$\begin{aligned}
 \angle C &= 180^\circ - (\angle A + \angle B) \\
 &\doteq 180^\circ - (45^\circ + 34.4^\circ) \\
 &\doteq 180^\circ - 79.4^\circ \\
 &\doteq 100.6^\circ
 \end{aligned}$$

Use the Sine Law to find c .

$$\begin{aligned}
 \frac{c}{\sin 100.6^\circ} &= \frac{30}{\sin 45^\circ} \\
 c &\doteq \frac{30 \sin 100.6^\circ}{\sin 45^\circ} \\
 &\doteq 41.7
 \end{aligned}$$

In $\triangle ABC$, $c \doteq 41.7$ cm, $\angle B \doteq 34.4^\circ$, and $\angle C \doteq 100.6^\circ$.

17. Draw a diagram.

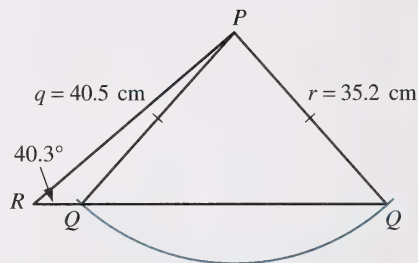
$$\begin{aligned}
 q \sin R &= 40.5 \sin 40.3^\circ \\
 &\doteq 26.2
 \end{aligned}$$

Because $q \sin R < r < q$, there are two locations for point Q , two triangles PQR , and two solutions.

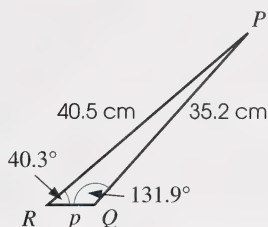
First, use the Sine Law to find the possible measures for $\angle PQR$.

$$\begin{aligned}
 \frac{\sin Q}{40.5} &= \frac{\sin 40.3^\circ}{35.2} \\
 \sin Q &= \frac{40.5 \sin 40.3^\circ}{35.2}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \angle Q &\doteq 48.1^\circ \quad \text{or} \quad \angle Q \doteq \quad^\circ - \quad^\circ \\
 &\doteq 131.9^\circ
 \end{aligned}$$



Solve both triangles PQR .

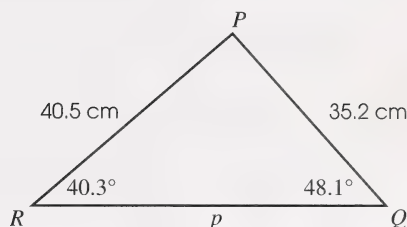


$$\begin{aligned}\angle P &= 180^\circ - (\angle R + \angle Q) \\ &\doteq 180^\circ - (40.3^\circ + 131.9^\circ) \\ &\doteq 180^\circ - 172.2^\circ \\ &\doteq 7.8^\circ\end{aligned}$$

Use the Sine Law to find p .

$$\begin{aligned}\frac{p}{\sin 7.8^\circ} &\doteq \frac{35.2}{\sin 40.3^\circ} \\ p &\doteq \frac{35.2 \sin 7.8^\circ}{\sin 40.3^\circ} \\ &\doteq 7.4\end{aligned}$$

In $\triangle PQR$, $\angle P \doteq 7.8^\circ$, $\angle Q \doteq 131.9^\circ$, and $p \doteq 7.4$ cm
or $\angle P \doteq 91.6^\circ$, $\angle Q \doteq 48.1^\circ$, and $p \doteq 54.4$ cm.



$$\begin{aligned}\angle Q &= 180^\circ - (\angle R + \angle P) \\ &\doteq 180^\circ - (40.3^\circ + 91.6^\circ) \\ &\doteq 180^\circ - 131.9^\circ \\ &\doteq 48.1^\circ\end{aligned}$$

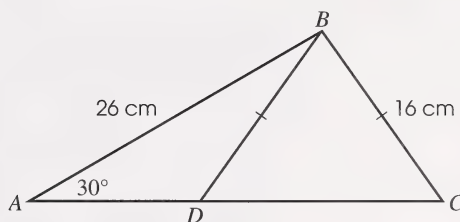
Use the Sine Law to find p .

$$\begin{aligned}\frac{p}{\sin 91.6^\circ} &\doteq \frac{35.2}{\sin 40.3^\circ} \\ p &\doteq \frac{35.2 \sin 91.6^\circ}{\sin 40.3^\circ} \\ &\doteq 54.4\end{aligned}$$

- 23. a.** Use the Sine Law to find $\angle BCD$.

$$\begin{aligned}\frac{\sin \angle BCD}{26} &= \frac{\sin 30^\circ}{16} \\ \sin \angle BCD &= \frac{26 \sin 30^\circ}{16} \\ \angle BCD &\doteq 54.3^\circ\end{aligned}$$

$$\begin{aligned}\therefore \angle BDA &= 180^\circ - \angle BCA \\ &\doteq 180^\circ - 54.3^\circ \\ &\doteq 125.7^\circ\end{aligned}$$



Section 2: Activity 2 (continued)

- b. Find
- $\angle CBD$
- .

$$\begin{aligned}
 \angle CBD &= 180^\circ - (\angle BDC + \angle BDC) \\
 &\doteq 180^\circ - (54.3^\circ + 54.3^\circ) \\
 &\doteq 180^\circ - 108.6^\circ \\
 &\doteq 71.4^\circ
 \end{aligned}$$

Use the Sine Law to find CD .

$$\begin{aligned}
 \frac{CD}{\sin 71.4^\circ} &\doteq \frac{16}{\sin 54.3^\circ} \\
 CD &\doteq \frac{16 \sin 71.4^\circ}{\sin 54.3^\circ} \\
 &\doteq 18.7
 \end{aligned}$$

The length of CD is approximately 18.7 cm.

- b. Textbook questions 31, 32, 34, and 35 of “Applications and Problem Solving,” pp. 511 and 512

31. Draw a diagram.

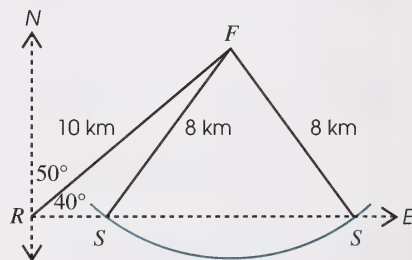
R is the location of the first ranger station; S is the location of the second ranger station; and F is the location of the fire.

Because $10 \sin 40^\circ \doteq 6.4 < 8 < 10$, there are two possible locations for S , two possible triangles, and two possible solutions.

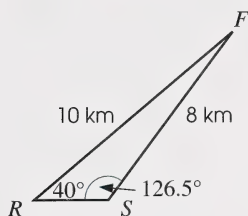
First, find $\angle FSR$.

$$\begin{aligned}
 \frac{\sin S}{10} &= \frac{\sin 40^\circ}{8} \\
 \sin S &= \frac{10 \sin 40^\circ}{8}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \angle S &\doteq 53.5^\circ \quad \text{or} \quad \angle S \doteq 180^\circ - 53.5^\circ \\
 &\doteq 126.5^\circ
 \end{aligned}$$

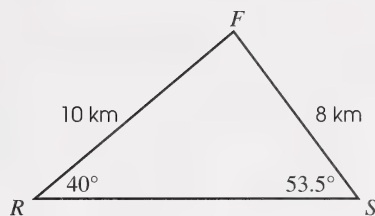


Solve both triangles.



$$\begin{aligned}\angle F &= 180^\circ - (\angle R + \angle S) \\ &\doteq 180^\circ - (40^\circ + 126.5^\circ) \\ &\doteq 180^\circ - 166.5^\circ \\ &\doteq 13.5^\circ\end{aligned}$$

$$\begin{aligned}\frac{RS}{\sin 13.5^\circ} &= \frac{8}{\sin 40^\circ} \\ RS &= \frac{8 \sin 13.5^\circ}{\sin 40^\circ} \\ &\doteq 3\end{aligned}$$

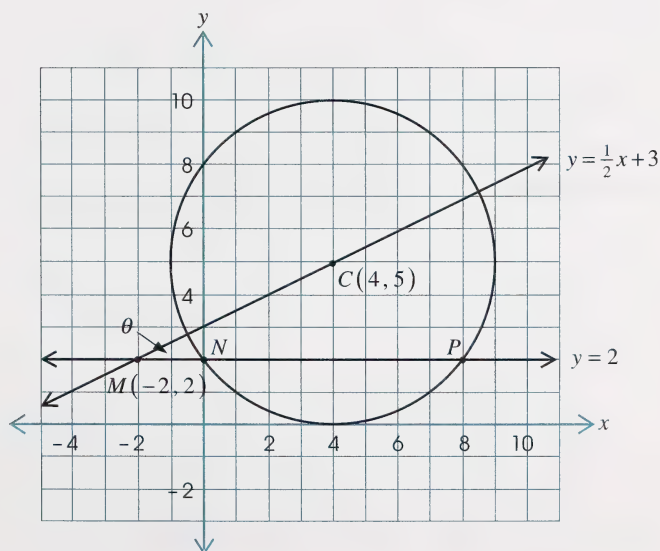


$$\begin{aligned}\angle F &= 180^\circ - (\angle R + \angle S) \\ &\doteq 180^\circ - (40^\circ + 53.5^\circ) \\ &\doteq 180^\circ - 93.5^\circ \\ &\doteq 86.5^\circ\end{aligned}$$

$$\begin{aligned}\frac{RS}{\sin 86.5^\circ} &= \frac{8}{\sin 40^\circ} \\ RS &= \frac{8 \sin 86.5^\circ}{\sin 40^\circ} \\ &= 12\end{aligned}$$

The stations are either about 3 km apart or about 12 km apart.

32. a. Draw a diagram.



Section 2: Activity 2 (continued)

Determine the points of intersection by solving the system of equations.

$$(x-4)^2 + (y-5)^2 = 25 \quad (1)$$

$$y = 2 \quad (2)$$

Substitute $y = 2$ into (1).

$$(x-4)^2 + (y-5)^2 = 25$$

$$(x-4)^2 + (2-5)^2 = 25$$

$$(x-4)^2 + 9 = 25$$

$$(x-4)^2 = 16$$

$$x-4 = \pm 4$$

$$\therefore x-4 = 4 \quad \text{or} \quad x-4 = -4$$

$$x = 8$$

$$x = 0$$

Therefore, the points of intersection are $N(0, 2)$ and $P(8, 2)$.

$$\begin{aligned} NP &= |x_2 - x_1| \\ &= |8 - 0| \\ &= 8 \end{aligned}$$

The length of chord NP is 8 units.

- b.** Before you can use the Sine Law to determine the length of chord NP , you must determine $\angle \theta$, the angle between $y = \frac{1}{2}x + 3$ and $y = 2$.

Now, the slope of $y = \frac{1}{2}x + 3$ is $\frac{1}{2}$ and $y = 2$ is 0.

Because $\text{slope} = \frac{\text{rise}}{\text{run}}$,

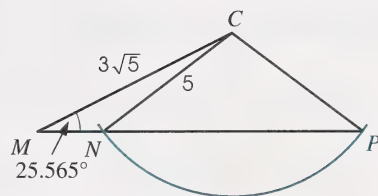
$$\begin{aligned} \frac{1}{2} &= \tan \theta \\ \angle \theta &\doteq 26.565^\circ \end{aligned}$$

Next, join CN and CP to form $\triangle CMN$ and $\triangle CMP$.

Use the distance formula to find the length of CM .

Let $(x_1, y_1) = (4, 5)$ and $(x_2, y_2) = (-2, 2)$.

$$\begin{aligned}
 CM &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2 - 4)^2 + (2 - 5)^2} \\
 &= \sqrt{(-6)^2 + (-3)^2} \\
 &= \sqrt{36 + 9} \\
 &= \sqrt{45} \\
 &= 3\sqrt{5}
 \end{aligned}$$



Also, the length of CN and CP is the radius of circle $(x - 4)^2 + (y - 5)^2 = 25$.

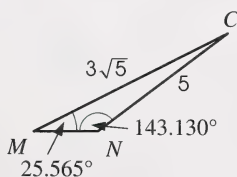
$$\therefore CN = CP = 5$$

Next, find $\angle CPN$ and $\angle CNM$.

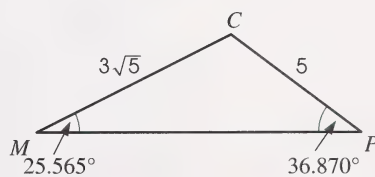
$$\begin{aligned}
 \frac{\sin P}{3\sqrt{5}} &= \frac{\sin 26.565^\circ}{5} \\
 \sin P &= \frac{3\sqrt{5} \sin 26.565^\circ}{5} \\
 \angle P &\doteq 36.870^\circ
 \end{aligned}$$

$$\begin{aligned}
 \therefore \angle CPN &\doteq 36.870^\circ \quad \text{and} \quad \angle CNM \doteq \quad \circ - \quad \circ \\
 &\doteq 143.130^\circ
 \end{aligned}$$

Use the Sine Law to find MN from $\triangle CMN$ and MP from $\triangle CMP$. The length of chord NP will be the difference between the lengths of MP and MN .



$$\begin{aligned}
 \angle MCN &\doteq 180^\circ - (26.565^\circ + 143.130^\circ) \\
 &\doteq 180^\circ - 169.695^\circ \\
 &\doteq 10.305^\circ
 \end{aligned}$$



$$\begin{aligned}
 \angle MCP &\doteq 180^\circ - (26.565^\circ + 36.870^\circ) \\
 &\doteq 180^\circ - 63.435^\circ \\
 &\doteq 116.565^\circ
 \end{aligned}$$

Section 2: Activity 2 (continued)

$$\begin{aligned}\frac{MN}{\sin 10.305^\circ} &\doteq \frac{5}{\sin 26.565^\circ} \\ MN &\doteq \frac{5 \sin 10.305^\circ}{\sin 26.565^\circ} \\ &\doteq 2.0\end{aligned}$$

$$\begin{aligned}\therefore NP &= MP - MN \\ &\doteq 10.0 - 2.0 \\ &\doteq 8.0\end{aligned}$$

The length of chord NP is about 8 units.

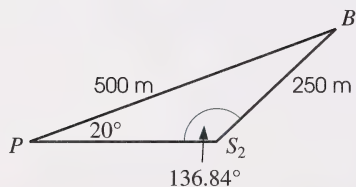
34. Draw a diagram.

Use the Sine Law to find $\angle BS_1P$ and $\angle BS_2P$.

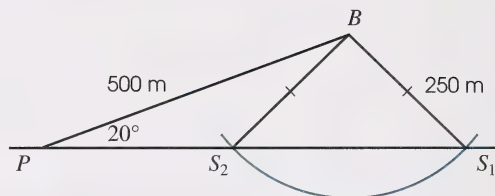
$$\begin{aligned}\frac{\sin S_1}{500} &= \frac{\sin 20^\circ}{250} \\ \sin S_1 &= \frac{500 \sin 20^\circ}{250}\end{aligned}$$

$$\begin{aligned}\therefore \angle BS_1P &\doteq 43.16^\circ \quad \text{and} \quad \angle BS_2P = 180^\circ - \angle BS_1P \\ &\doteq 180^\circ - 43.16^\circ \\ &\doteq 136.84^\circ\end{aligned}$$

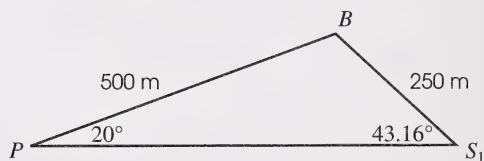
Solve $\triangle BPS_2$ for PS_2 .



$$\begin{aligned}\angle PBS_2 &= 180^\circ - (\angle P + \angle S_2) \\ &\doteq 180^\circ - (20^\circ + 136.84^\circ) \\ &\doteq 180^\circ - 156.84^\circ \\ &\doteq 23.16^\circ\end{aligned}$$



Solve $\triangle BPS_1$ for PS_1 .



$$\begin{aligned}\angle PBS_1 &= 180^\circ - (\angle P + \angle S_1) \\ &\doteq 180^\circ - (20^\circ + 43.16^\circ) \\ &\doteq 180^\circ - 63.16^\circ \\ &\doteq 116.84^\circ\end{aligned}$$

$$\frac{PS_2}{\sin 23.16^\circ} \doteq \frac{250}{\sin 20^\circ}$$

$$PS_2 \doteq \frac{250 \sin 23.16^\circ}{\sin 20^\circ}$$

$$\doteq 287.51$$

$$\frac{PS_1}{\sin 116.84^\circ} \doteq \frac{250}{\sin 20^\circ}$$

$$PS_1 \doteq \frac{250 \sin 116.84^\circ}{\sin 20^\circ}$$

$$\doteq 652.2$$

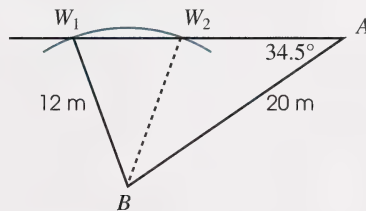
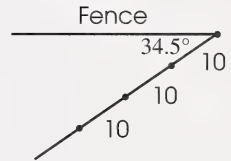
$$\therefore PS_1 - PS_2 \doteq 652.2 - 287.5$$

$$\doteq 365$$

The length of illuminated shoreline is approximately 365 m.

35. First, determine whether water from the second sprinkler reaches the fence.

Because $20 \sin 34.5^\circ \doteq 11.328$ m, and the range of each sprinkler is 12 m, water from the second sprinkler reaches the fence. However, the third sprinkler is too far away!



The portion of the fence that gets wet by the second sprinkler extends from W_1 to W_2 .

In order to determine the distance W_1 lies from A, you need to find $\angle W_1$ and $\angle B$.

$$\frac{\sin W_1}{20} = \frac{\sin 34.5^\circ}{12}$$

$$\angle W_1 \doteq 70.7^\circ$$

$$\angle B = 180^\circ - (\angle A + \angle W_1)$$

$$\doteq 180^\circ - (34.5^\circ + 70.7^\circ)$$

$$\doteq 180^\circ - 104.5^\circ$$

$$\doteq 74.8^\circ$$

Use the Sine Law to determine the distance W_1 lies from A.

$$\frac{W_1 A}{\sin 74.8^\circ} \doteq \frac{12}{\sin 34.5^\circ}$$

$$W_1 A \doteq \frac{12 \sin 74.8^\circ}{\sin 34.5^\circ}$$

$$\doteq 20.4$$

Therefore, W_1 lies approximately 20.4 m from A.

Section 2: Activity 2 (continued)

Next, determine if W_2 is within the range of the first sprinkler. If it is, then the entire 20.4 m of fence from W_1 to A will be wet.

Use the Sine Law to determine the distance W_2 lies from A.

$$\angle BW_2A = 180^\circ - \angle BW_1A$$

$$\doteq 180^\circ - 70.7^\circ$$

$$\doteq 109.3^\circ$$

$$\angle ABW_2 = 180^\circ - (\angle A + \angle BW_2A)$$

$$\doteq 180^\circ - (34.5^\circ + 109.3^\circ)$$

$$\doteq 180^\circ - 143.8^\circ$$

$$\doteq 36.2^\circ$$

$$\frac{W_2A}{\sin 36.2^\circ} \doteq \frac{12}{\sin 34.5^\circ}$$

$$W_2A \doteq \frac{12 \sin 36.2^\circ}{\sin 34.5^\circ}$$

$$\doteq 12.5$$

Therefore, W_2 lies approximately 12.5 m from A.

Determine the farthest the first sprinkler reaches to the left.

$$\frac{\sin \theta_2}{10} = \frac{\sin 34.5^\circ}{12}$$

$$\sin \theta_2 = \frac{10 \sin 34.5^\circ}{12}$$

$$\angle \theta_2 \doteq 28.2^\circ$$

$$\angle \theta_1 = 180^\circ - (\angle A + \angle \theta_2)$$

$$\doteq 180^\circ - (34.5^\circ + 28.2^\circ)$$

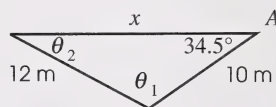
$$\doteq 180^\circ - 62.7^\circ$$

$$\doteq 117.3^\circ$$

$$\therefore \frac{x}{\sin \theta_1} = \frac{12}{\sin 34.5^\circ}$$

$$x \doteq \frac{12 \sin 117.3^\circ}{\sin 34.5^\circ}$$

$$\doteq 18.8$$



Therefore, the first sprinkler reaches 18.8 m to the left.

Because $18.8 \text{ m} > 12.5 \text{ m}$, the portion of the fence that gets wet by the first sprinkler overlaps the portion of the fence that gets wet by the second sprinkler. Therefore, 20.4 m of fence gets wet.

3. Anna's position is point A. The tee is at T.

Use the Sine Law to find $\angle B_2$.

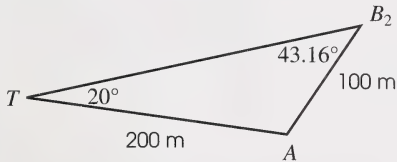
$$\frac{\sin B_2}{200} = \frac{\sin 20^\circ}{100}$$

$$\sin B_2 = \frac{200 \sin 20^\circ}{100}$$

$$\angle B_2 \doteq 43.16^\circ$$

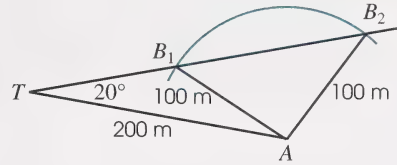
$$\begin{aligned}\therefore \angle TB_1A &= 180^\circ - \angle B_2 \\ &\doteq 180^\circ - 43.16^\circ \\ &\doteq 136.84^\circ\end{aligned}$$

Solve $\triangle TAB_2$ for TB_2 .

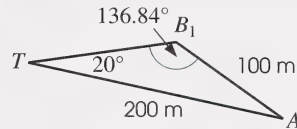


$$\begin{aligned}\angle TAB_2 &= 180^\circ - (\angle T + \angle B_2) \\ &\doteq 180^\circ - (20^\circ + 43.16^\circ) \\ &\doteq 180^\circ - 63.16^\circ \\ &\doteq 116.84^\circ\end{aligned}$$

$$\begin{aligned}\therefore \frac{TB_2}{\sin 116.84^\circ} &\doteq \frac{100}{\sin 20^\circ} \\ TB_2 &\doteq \frac{100 \sin 116.84^\circ}{\sin 20^\circ} \\ &\doteq 261\end{aligned}$$



Solve $\triangle TAB_1$ for TB_1 .



$$\begin{aligned}\angle TAB_1 &= 180^\circ - (\angle T + \angle B_1) \\ &\doteq 180^\circ - (20^\circ + 136.84^\circ) \\ &\doteq 180^\circ - 156.84^\circ \\ &\doteq 23.16^\circ\end{aligned}$$

$$\begin{aligned}\therefore \frac{TB_1}{\sin 23.16^\circ} &= \frac{100}{\sin 20^\circ} \\ TB_1 &\doteq \frac{100 \sin 23.16^\circ}{\sin 20^\circ} \\ &\doteq 115\end{aligned}$$

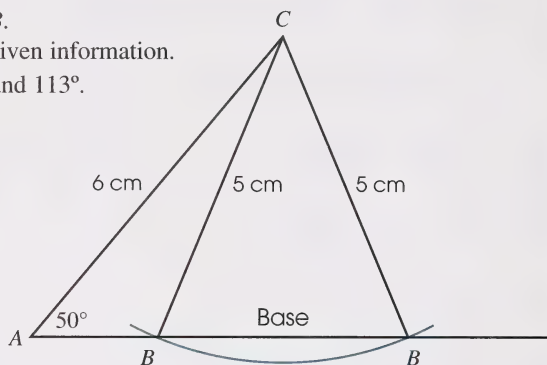
The ball could be approximately 261 m from the tee or approximately 115 m from the tee.

Section 2: Follow-up Activities

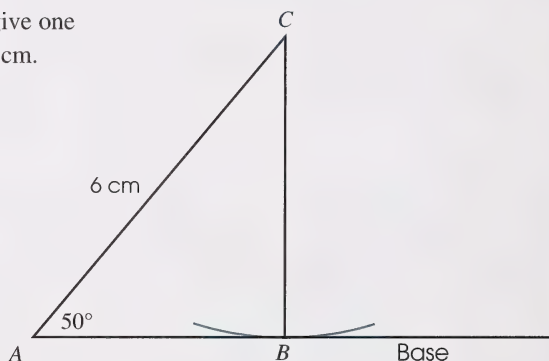
Extra Help

1. Textbook questions 1 to 3 of Investigation 1, “Exploring the SSA Case,” p. 500

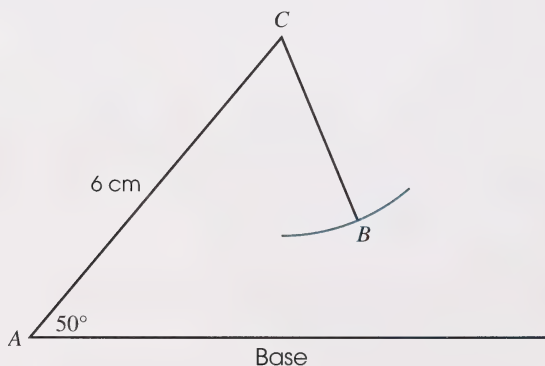
1. a. There are two possible locations for point B .
 b. You can construct two triangles using the given information.
 c. The approximate values of $\angle CBA$ are 67° and 113° .



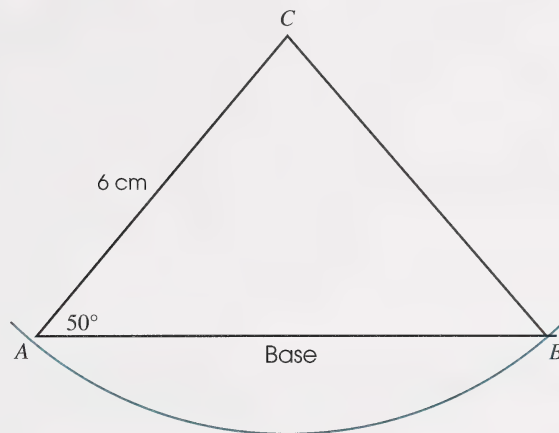
2. a. The approximate length of CB that would give one location for point B , and one $\triangle ABC$, is 4.6 cm.
 b. The approximate measure of $\angle B$ is 90° .



3. a. If CB is less than 4.6 cm, no triangle can be drawn.



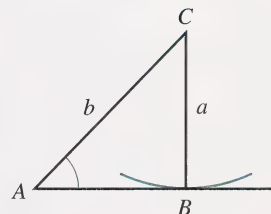
- b. If $CB \geq CA$, exactly one triangle can be drawn.



2. Textbook questions 1 to 3 of Investigation 2, “Making Generalizations,” p. 501

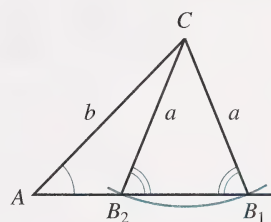
1. a. If $a < b$ and there is one position for point B and one $\triangle ABC$, then $\angle ABC = 90^\circ$. The shortest distance from a point to a line is the perpendicular distance.

b. $\sin A = \frac{a}{b}$
 $a = b \sin A$



2. a. If the length of a yields two triangles, then the measures of $\angle ABC$ in the two triangles are supplementary; that is, the two possible values of $\angle ABC$ add up to 180° . $\triangle CB_2B_1$ is isosceles and $\angle CB_2B_1 = \angle CB_1B_2$.

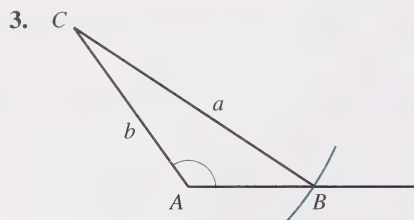
$$\therefore \angle CB_2A = 180^\circ - \angle CB_1A$$



b. In each triangle, $\frac{a}{\sin A} = \frac{b}{\sin B}$.

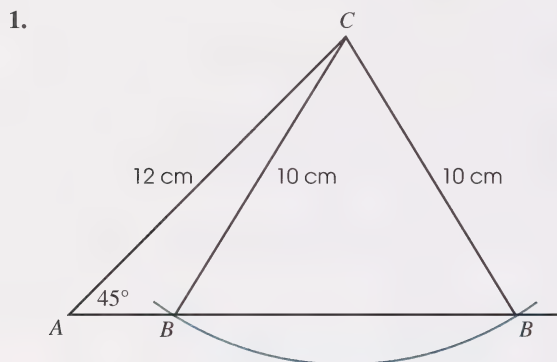
- c. If there are two possible locations for B , then $b \sin A < a < b$.

Extra Help (continued)

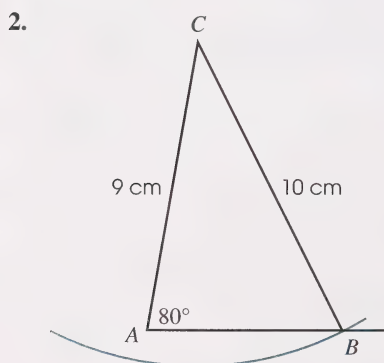


- a. If one $\triangle ABC$ exists, then $a > b$.
- b. If no $\triangle ABC$ exists, then $a < b$.

3. Textbook questions 1 to 3 of Investigation 3, “Applying the Concepts,” p. 501

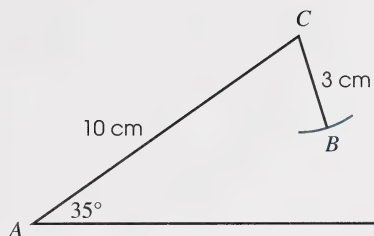


Because $12 \sin 45^\circ \doteq 8.5 \text{ cm} < 10 \text{ cm} < 12 \text{ cm}$, there are two possible locations for point B and two possible triangles ABC .



Because $CB > AC$, there is only one possible location for point B and one possible $\triangle ABC$.

3.



Because $10 \sin 35^\circ \doteq 5.7$ cm is less than CB , no $\triangle ABC$ is possible.

Enrichment

1. Textbook questions 1 to 4 of Investigation 1, “Finding the Third Side,” p. 513

$$\begin{aligned}
 1. \quad a. \quad & a^2 = b^2 + c^2 - 2bc \cos A \\
 & (5\sqrt{3})^2 = 10^2 + c^2 - 2(10)c \cos 60^\circ \\
 & 25(3) = 100 + c^2 - 20c(0.5) \\
 & 75 = 100 + c^2 - 10c \\
 & 0 = c^2 - 10c + 25 \\
 & 0 = (c - 5)^2 \\
 & c - 5 = 0 \\
 & c = 5
 \end{aligned}$$

Therefore, c is 5 cm.

b. Only one triangle is possible because there is a single value of c , the third side.

Enrichment (continued)

2. a. $a^2 = b^2 + c^2 - 2bc \cos A$

$$9^2 = 10^2 + c^2 - 2(10)c \cos 60^\circ$$

$$81 = 100 + c^2 - 20c(0.5)$$

$$81 = c^2 - 10c + 100$$

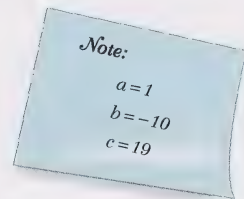
$$0 = c^2 - 10c + 19$$

$$\begin{aligned} \therefore c &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(19)}}{2(1)} \\ &= \frac{10 \pm \sqrt{100 - 76}}{2} \\ &= \frac{10 \pm \sqrt{24}}{2} \\ &= \frac{10 \pm 2\sqrt{6}}{2} \\ &= \frac{2(5 \pm \sqrt{6})}{2} \\ &= 5 \pm \sqrt{6} \end{aligned}$$

$$\begin{aligned} \therefore c &= 5 + \sqrt{6} \quad \text{or} \quad c = 5 - \sqrt{6} \\ &\doteq 7.4 \quad \quad \quad \doteq 2.6 \end{aligned}$$

Therefore, c is approximately 7.4 cm or 2.6 cm.

- b. Because there are two possible values for the third side, two triangles are possible.



3. a. $a^2 = b^2 + c^2 - 2bc \cos A$

$$12^2 = 10^2 + c^2 - 2(10)c \cos 60^\circ$$

$$144 = 100 + c^2 - 20c(0.5)$$

$$144 = c^2 - 10c + 100$$

$$0 = c^2 - 10c - 44$$

$$\begin{aligned} \therefore c &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-44)}}{2(1)} \\ &= \frac{10 \pm \sqrt{100 + 176}}{2} \\ &= \frac{10 \pm \sqrt{276}}{2} \\ &= \frac{10 \pm 2\sqrt{69}}{2} \\ &= \frac{2(5 \pm \sqrt{69})}{2} \\ &= 5 \pm \sqrt{69} \end{aligned}$$

$$\begin{aligned} \therefore c &= 5 + \sqrt{69} \quad \text{or} \quad c = 5 - \sqrt{69} \\ &\doteq 13.3 \quad \quad \quad \doteq -3.3 \end{aligned}$$

Impossible!

Therefore, c is approximately 13.3 cm.

- b. Because the length of the third side must be positive, only one solution and, therefore, one triangle is possible.

Note:

$$\begin{aligned} a &= 1 \\ b &= -10 \\ c &= -44 \end{aligned}$$

Enrichment (continued)

$$\begin{aligned}
 4. \quad a. \quad & a^2 = b^2 + c^2 - 2bc \cos A \\
 & 8^2 = 10^2 + c^2 - 2(10)c \cos 60^\circ \\
 & 64 = 100 + c^2 - 20c(0.5) \\
 & 64 = c^2 - 10c + 100 \\
 & 0 = c^2 - 10c + 36
 \end{aligned}$$

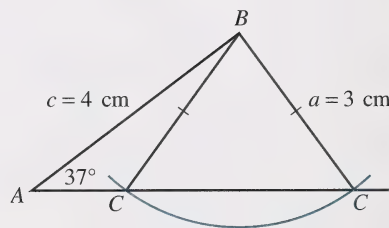
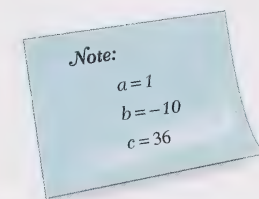
$$\begin{aligned}
 \therefore c &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(36)}}{2(1)} \\
 &= \frac{10 \pm \sqrt{100 - 144}}{2} \\
 &= \frac{10 \pm \sqrt{-44}}{2}
 \end{aligned}$$

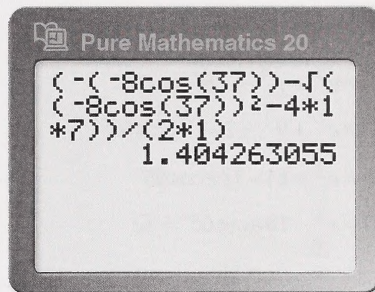
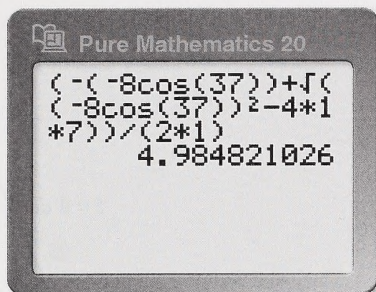
Therefore, c is non-real.

b. Because the values of c are non-real, no triangle is possible.

2. Textbook questions 1 and 2 of Investigation 2, “Solving Triangles,” p. 513

$$\begin{aligned}
 1. \quad a. \quad & a^2 = b^2 + c^2 - 2bc \cos A \\
 & 3^2 = b^2 + 4^2 - 2b(4) \cos 37^\circ \\
 & 9 = b^2 + 16 - 8b \cos 37^\circ \\
 & 0 = b^2 - 8b \cos 37^\circ + 7 \\
 \therefore b &= \frac{-(-8 \cos 37^\circ) \pm \sqrt{(-8 \cos 37^\circ)^2 - 4(1)(7)}}{2(1)}
 \end{aligned}$$

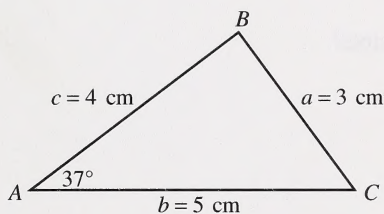




$$\therefore b = 5.0 \text{ cm} \quad \text{or} \quad 1.4 \text{ cm}$$

Because there are two values of b , two triangles are possible.

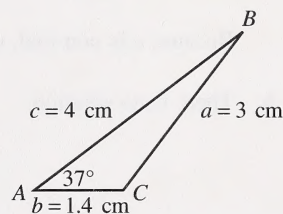
- b. Find $\angle B$ for both triangles. **Note:** Use the calculator values for b .



$$\begin{aligned} \frac{\sin B}{4.984821026} &\doteq \frac{\sin 37^\circ}{3} \\ \sin B &\doteq \frac{4.984821026 \sin 37^\circ}{3} \\ \angle B &\doteq 89.6^\circ \end{aligned}$$

$$\begin{aligned} \therefore \angle C &= 180^\circ - (\angle A + \angle B) \\ &\doteq 180^\circ - (37^\circ + 89.6^\circ) \\ &\doteq 180^\circ - 126.6^\circ \\ &\doteq 53.4^\circ \end{aligned}$$

In $\triangle ABC$, $b \doteq 5.0 \text{ cm}$, $\angle B \doteq 89.6^\circ$, and $\angle C \doteq 53.4^\circ$
or $b = 1.4 \text{ cm}$, $\angle B \doteq 16.4^\circ$, and $\angle C = 126.6^\circ$



$$\begin{aligned} \frac{\sin B}{1.404263055} &\doteq \frac{\sin 37^\circ}{3} \\ \sin B &\doteq \frac{1.404263055 \sin 37^\circ}{3} \\ \angle B &\doteq 16.4^\circ \end{aligned}$$

$$\begin{aligned} \therefore \angle C &= 180^\circ - (\angle A + \angle B) \\ &\doteq 180^\circ - (37^\circ + 16.4^\circ) \\ &\doteq 180^\circ - 53.4^\circ \\ &\doteq 126.6^\circ \end{aligned}$$

Enrichment (continued)

$$\begin{aligned}
 2. \quad a. \quad d^2 &= e^2 + f^2 - 2ef \cos D \\
 7^2 &= e^2 + 9^2 - 2e(9)\cos 65^\circ \\
 49 &= e^2 + 81 - 18e \cos 65^\circ \\
 0 &= e^2 - 18e \cos 65^\circ + 32
 \end{aligned}$$

$$\begin{aligned}
 \therefore e &= \frac{-(-18 \cos 65^\circ) \pm \sqrt{(-18 \cos 65^\circ)^2 - 4(1)(32)}}{2(1)} \\
 &= \frac{7.61 \pm \sqrt{57.87 - 128}}{2} \\
 &= \frac{7.61 \pm \sqrt{-70.13}}{2}
 \end{aligned}$$

Because e is non-real, no triangle is possible!

b. There is no solution.

